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A critical assessment of methods for the determination of the shear stress amplitude in multiaxial fatigue criteria belonging to critical plane class

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A B S T R A C T

Multiaxial high cycle fatigue criteria based on the critical plane approach necessitate unambiguous definitions of the amplitude and mean value of the shear stress (τ_a and τ_m) acting on the material planes. Four of the existing definitions relate the values of τ_a and τ_m to a geometrical element of the curve described by the tip of the shear stress vector (curve Ψ), respectively, the radius of the Minimum Circumscribed Circle, the Longest Chord, the Longest Projection, the diagonal of the Maximum Rectangular Hull (MRH).

In this paper a critical assessment of the above definitions is proposed, focusing on that based on the concept of MRH, which is the most recently developed. The main issues of the comparison are the uniqueness of the solution in the determination of τ_a and τ_m, the ability to differentiate proportional and non-proportional stresses, the differences of the values of τ_a obtained by each of the 4 methods for differently shaped curves Ψ.

1. Introduction

Multiaxial high cycle fatigue criteria based on the critical plane approach require the determination of the shear stress amplitude τ_a acting onto the planes passing through the analyzed material point [1–7]. The shear stress amplitude τ_a relative to a plane Δ, identified by its unit normal vector $\mathbf{n}=[n_x, n_y, n_z]^T$, has to be evaluated by processing the values of the components of the shear stress vector acting onto the plane, $\boldsymbol{\tau}_n(t)$ (Fig. 1a). The critical plane is determined as that experiencing the maximum shear stress amplitude $\tau_{a_{max}}$ [2,4–6], or the maximum value of a parameter involving τ_a and the normal stress $\boldsymbol{\sigma}_n(t)$ acting onto the plane (Fig. 1a) [1,3].

In many cases, multiaxial fatigue criteria are used for cyclic states of stress, for which the components of the stress tensor assume the same values after a time period *T*. In the case of proportional stresses, the $\boldsymbol{\tau}_n$ vector maintains a constant direction and its tip describes a straight line segment (Fig. 1b); in this case, the determination of τ_a and of the mean component of the shear stress, τ_m, can be obtained directly from the maximum and minimum values of the modulus of $\boldsymbol{\tau}_n(t)$. In the more general case of non-proportional stresses, both direction and magnitude of $\boldsymbol{\tau}_n(t)$ vary with time, so that, in the time period *T*, the tip of the shear stress vector draws an imaginary closed curve Ψ, as in the example of Fig. 1 [4,5,8]. In this case the identification and the consequent evaluation of τ_a and τ_m is far from trivial and various definitions have been proposed [4,5,8–10,12–21].

In the comparison carried out in this paper four definitions of τ_a and τ_m have been mainly considered; each of them relates the value of τ_a and τ_m to a geometrical element of the curve Ψ, in particular the Minimum Circumscribed Circle (MCC), the Longest Chord (LC), the Longest Projection (LP), the Maximum Rectangular Hull (MRH). Some considerations about the definitions related to the Minimum Circumscribed Ellipse (MCE) and the Minimum Circumscribed Rectangle (MCR) in comparison to that of the MRH are also reported.

In all the definitions, the value of τ_a is related to a parameter representative of the extent of the geometrical element, while τ_m is related to a particular point in the u–v plane, whose position depends on the position and the shape of the curve Ψ.

The Minimum Circumscribed Circle (MCC) to a curve is the smallest circle that contains all the points of the curve, as shown in Fig. 2a, where *R* is the radius of the MCC and *C* its center. According to the MCC method [4,5,8,9,12–15], τ_a is equal to *R* and τ_m is equal to the modulus of the vector joining point *O* to point *C*.

The Longest Chord of a curve is the segment joining the two farthest points one to the other among all the possible pairs of points of the curve, as shown in Fig. 2b where *A*, *B* and C_C are the extremes

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Nomenclature

t	time
T	time period
N	number of instants
Δ	generic material plane
$\mathbf{n}$	unit normal vector of the plane Δ
$\boldsymbol{\sigma}_n(t)$	normal component of the stress vector acting onto a generic plane
$\tau_n(t)$	shear component of the stress vector acting onto a generic plane
τ_a	shear stress amplitude
τ_m	mean values of shear stress
$\tau_{a,c}$	shear stress amplitude in the critical plane
$\tau_{a_{\max}}$	maximum shear stress amplitude in a material point
Ψ	curve described by the tip of the $\tau_n(t)$ vector
τ_u, τ_v	coordinates of the points of the continuous curve Ψ
τ_{u_i}, τ_{v_i}	coordinates of the points of the curve Ψ at the time instants t_i
MCC	Minimum Circumscribed Circle
LC	Longest Chord
LP	Longest Projection
MRH	Maximum Rectangular Hull
OP	Orthogonal Projection (projection of the curve Ψ along a line orthogonal to the LP)
LSRH	Longest Size Rectangular Hull
MCE	Maximum Circumscribed Ellipse
MCR	Minimum Circumscribed Rectangle
R	radius of the MCC
L_C	half-length of the LC
L_P	half-length of the LP
L_H	half-length of the diagonal MRH
L_O	half-length of the OP
R_a	length of the major semi-axis of the MCE
R_b	length of the minor semi-axis of the MCE
C	center of the MCC
A, B	endpoints of the LC
C_C	midpoint of the LC
C_P	midpoint of the LP
C_H	center of the MRH
U, V	coordinates of C
U_g, V_g	coordinates of a generic point of the plane Δ
r_i	distances of the points of the curve from C
$d_{i,j}$	distance between the i-th and the j-th point of the curve Ψ
$d_{A,B}$	distance between points A and B (length of the LC)
γ	angle between the x and u axes
$L_x(\gamma)$	half-length of the projection of the curve Ψ along x direction
$L_y(\gamma)$	half-length of the projection of the curve Ψ along y direction
$L_H(\gamma)$	half-lengths of the diagonals of the circumscribed hulls of the curve Ψ
$\tau_{zx,a}(\gamma)$	amplitude of the shear stress acting in the x direction, coincident with $L_x(\gamma)$)
$\tau_{zy,a}(\gamma)$	amplitude of the shear stress acting in the y direction, coincident with $L_y(\gamma)$
$\tau_{z,a}(\gamma)$	amplitude of the equivalent shear stress, coincident with $L_H(\gamma)$
Γ_{LP}	angle of the maximum value of the function $L_x(\gamma) = \tau_{zx,a}(\gamma)$
Γ_{LH}	angle of the maximum value of the function $L_H(\gamma) = \tau_{z,a}(\gamma)$

and the midpoint of the LC, $d_{A,B}$ and $L_C = d_{A,B}/2$ are the length and the half-length of the LC, P_O and $L_O = P_O/2$ are the length and the half-length of the Projection of the curve along a line orthogonal to the direction of the LC segment (OP). According to the LC method [9,10] τ_a is equal to L_C, and τ_m is equal to the modulus of the vector joining point O to point C_C.

The Longest Projections (LP) is the longest of the line segment obtained by projecting the curve Ψ on every line of the plane Δ, emanating from the origin O [9,11], as shown in Fig. 2c, where P_L and $L_P = P_L/2$ are respectively the length and the half-length of the LP. According to the LP method τ_a is equal to L_P and τ_m is equal to the modulus of the vector joining point O to point C_P.

The Maximum Rectangular Hull (MRH) to a curve is the rectangle with the longest diagonal among all those containing all the points of the curve and whose four sides are in contact with the curve itself, at least in one point, as shown in Fig. 2d, where L_H is the half-length of the diagonal of the MRH and C_H is its center. According to the MRH method [16] τ_a is equal to L_H, while no definition has been provided for τ_m [16,17].

In the MCE approach [18–20] the smallest ellipse that circumscribes the curve Ψ is properly determined, then τ_a is evaluated as the square root of the sum of the squares of the lengths of its major and minor semi-axis, R_a and R_b. The method based on the MCR [21] is similar to that of the MRH, being the minimum rectangular hull considered rather than the maximum one.

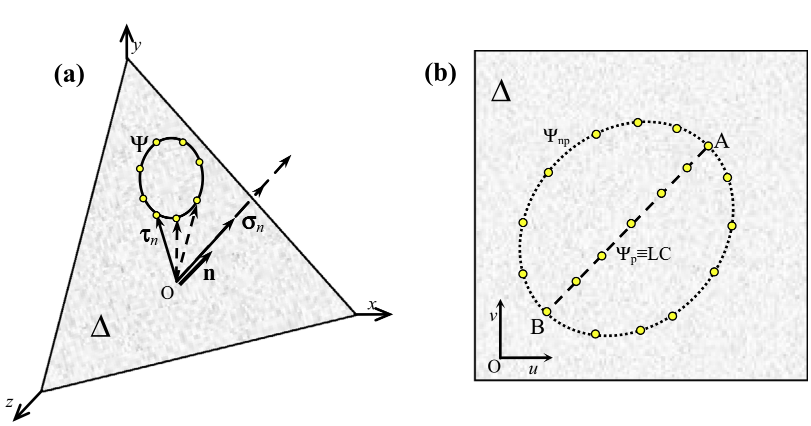

Fig. 1. Components of the stress vector acting onto a generic material plane Δ: (a) σ_n and τ_n components; (b) the curves Ψ described by the tip of τ_n in the case of proportional (straight line segment Ψ_p) and non-proportional (Ψ_{np}) stress.