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Analysis of Ultra Low Cycle Fatigue problems with the Barcelona plastic damage model and a new isotropic hardening law

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ABSTRACT

This paper presents a plastic-damage formulation and a new isotropic hardening law, based on the Barcelona plastic damage model initially proposed by Lubliner et al. (1989) [1], which is capable of predicting steel failure due to Ultra Low Cycle Fatigue (ULCF). This failure mechanism is obtained when the material is subjected to cyclic loads and breaks after applying a very low number of cycles, usually less than hundreds. The failure is driven by the plastic response of the material, and it is often predicted based on the plastic strains applied to it. The model proposed in this work has been formulated with the objective of predicting accurately the plastic behavior of the material, as well as its failure due to ULCF. This is achieved taking into account the fracture energy dissipated during the whole loading process. This approach allows the simulation of ULCF when it takes place due to regular cyclic loads or non-regular cyclic loads, as it is the case of seismic loads. Several simulations are conducted in order to show the capabilities of the formulation to reproduce the mechanical response of steel when it is subjected to regular and non-regular cyclic loads. The formulation is validated comparing the numerical results with several experimental tests made on X52 steel specimens. The agreement between the numerical and experimental results asses the validity of the proposed model to predict the plastic behavior of steel and its failure due to Ultra Low Cycle Fatigue.

1. Introduction

The mechanical phenomenon known as fatigue consists in the loss of material strength, and consequent failure, due to the effect of cyclic loads. Fatigue is characterized, among other parameters, by the number of cycles, load amplitude and reversion index [2–4]. Material failure is produced by an inelastic behavior, micro-cracking and crack coalescence, which lead to the final collapse of structural parts.

The fatigue phenomenon is defined more generally in the ASTM E1823 standard as: *"the process of permanent, progressive and localized structural change which occurs to a material point subjected to strains and stresses of variable amplitudes which produce cracks which lead to total failure after a certain number of cycles"* [5]. In this definition it is possible to include all fatigue ranges, from "Ultra Low Cycle Fatigue" (ULCF), to "Low Cycle Fatigue" (LCF) and "High Cycle Fatigue" (HCF).

While there is a general agreement that for failures in the range of 10^6 to 10^8 cycles the structure has failed in the high cycle fatigue range, there is not such agreement in defining the limits for low cycle and Ultra Low Cycle Fatigue. Authors such as Kanvinde and Deierlein [6] consider that LCF is found between 100 s and 1000 s cycles and that ULCF is in the range of 10–20 cycles; and other authors, such as Xue [7], put these limits in 10^4 for LCF and 100 for ULCF. However, despite these discrepancies, there is a general agreement that plastic behavior of the material plays an important role in the failure due to LCF or ULCF [8].

According to the literature review made by Yao and Munse in [9], first attempts to characterize LCF and ULCF can be attributed to Kommers who, in 1912, conducted several tests on a cantilever specimen subjected to cyclic bending. After these tests he reached the conclusion that the magnitude of deflection plays an important role in low cycle fatigue. However, main efforts to characterize the parameters driving LCF and ULCF are not found until 1950s, when numerous experimental programs where carried out to calibrate

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the material constants for various metals. A large amount of work is documented from this period. The experimental data is usually plotted on a log–log scale with the abscissa representing the number of life cycles and the ordinate the plastic strain amplitude. This graph is known as the $\Delta\varepsilon^p$–N curve. Following this approach, probably the most know, and most widely used, procedure to predict material failure under LCF and ULCF is the Manson–Coffin law [8,10,11]:

$$\Delta\varepsilon_p \cdot N^\alpha = C \tag{1}$$

$\Delta\varepsilon_p$ being the plastic strain range in the material, N the number of cycles that can be applied before ULCF and LCF failure, and α and C material constants.

From this first equation proposed by Coffin and Manson, several authors have provided their own law in order to improve the accuracy on the predicted cycles before failure, especially in the Ultra Low Cycle Fatigue regime. For instance, Xue [7] observed, from experimental results, that the law did not fit well in the range of very low life cycles, less than 100, so he proposed a new exponential damage rule that improved this accuracy. Kuroda [11] also provided a modification on the original Coffin–Manson law in order to predict the failure below 100 cycles. In this case the model is based on the accumulation of damage due to three different effects: tensile straining, cyclic straining and crack propagation.

The approach used by Tateishi et al. [12] to simulate LCF failure is also interesting. These authors use Miner's rule to couple the effect of high cycle fatigue with the effect of low cycle fatigue, by adding a ductile damage term. This last term depends on the yield strain of the material, the rupture strain and the strain that is applied in a given cycle.

One of the main drawbacks of most of the existing formulations to characterize ULCF and LCF is that they require regular cycles to predict material failure, or they couple the effects of non-regular cycles using the Miner's rule, which requires knowing the performance of the structure under regular cycles. However, this regularity often does not exist. An example of an ULCF failure due to an irregular cyclic load is found in the failure of structures subjected to seismic loads, where the frequency varies along time and each cycle may have different amplitudes.

An interesting approach to characterize low cycle fatigue accounting for non-regular cycles is the one proposed by Jiang et al. [13], which defines an independent continuous cumulative damage function (EVICD) based on the accumulation of plastic strain energy. This formulation is based on previous models of EVICD [14–16] and states that the total damage can be computed as:

$$D = \int dD \quad \text{with} \quad dD = \zeta \cdot dW^p \tag{2}$$

being D the fatigue damage, W^p the plastic strain energy density and ζ a function determined experimentally based on the fatigue response of the material. With this approach, the authors obtain an evolution of the fatigue damage parameter as the simulation evolves, the material failure is obtained when $D = 1$. In [13], the model is tested for fatigue ranges between 10^3 and 10^7 cycles, which corresponds to low and high cycle fatigue.

Another interesting approach based on damage accumulation is the one proposed by Kanvinde and Deierlein [6,17,18]. These authors, in order to account for the effects of void growth and coalescence that drive the fracture of metallic materials, propose a model that calculates the void growth and compares it with a critical value to detect material failure. This parameter is obtained experimentally. The initial formulation developed for monotonic cases (Void Growth Model – VGM [17]) is extended to cyclic loads by differentiating the void growth obtained in the tensile and compressive regions of the load cycle. Therefore, the void growth in the Cyclic Void Growth Model (CVGM) can be obtained as [18]:

$$VGI_{cyclic} = \sum_{tensile\ cycles} C_1 \cdot \int_{\varepsilon_1}^{\varepsilon_2} \exp\left(\left|1.5\frac{\sigma_m}{\sigma_e}\right|\right) d\varepsilon_p - \sum_{compressive\ cycles} C_2 \cdot \int_{\varepsilon_1}^{\varepsilon_2} \exp\left(\left|1.5\frac{\sigma_m}{\sigma_e}\right|\right) d\varepsilon_p < VGI_{cyclic}^{critical} \tag{3}$$

This formulation, as well as the formulation proposed by Jiang et al. [13], are capable to account for regular and non-regular cycles, as both formulations are based on the addition of certain quantities while the material increases its plastic strain. However, they both have the drawback of being based on a failure criterion that is completely independent of the plastic model (uncoupled approaches): it is calculated as the simulation advances and, when it reaches a certain level, the criterion tells the code that the material has failed.

The simulation of LCF and ULCF has also been approached using non-linear constitutive laws. This is the case of Saanouni and Abdul-Latif [19,20], who propose the use of a representative volume element (RVE), and a non-linear law based on the slip theory, to account for the dislocation movement of metallic grains. Instead of a RVE, Naderi et al. [21] proposed simulating the progressive failure of a given structural element by applying random properties to the different finite elements in which it is discretized. The constitutive model used to characterize LCF failure is the one defined by Lemaitre and Chaboche in [22]. The use of a stochastic approach is also the approach used by Warhadpande et al. [23], who applied random properties to a Voronoi cell. In most of these models the damage variable is also calculated independently of the non-linear constitutive law used to simulate the material performance.

Current work proposes the use of a plastic damage model, and presents a new isotropic hardening law, to simulate Ultra Low Cycle Fatigue. The model developed is based on the Barcelona model originally formulated by Lubliner et al. [1,24–26]. Although this model was originally defined to simulate brittle materials such as concrete, here is used with a kinematic and isotropic hardening law specifically defined for the simulation of steel. The isotropic hardening law is defined with an initial hardening region followed by a softening region. One of the main characteristics of the model is that the isotropic hardening behavior of the material is driven by the plastic energy dissipated: the model measures the fracture energy that is dissipated as the plastic strain increases, and this energy is used to define the plastic strain level at which material softening due to damage starts and finishes. The model considers that damage initiates when the plastic law reaches the softening region and the complete failure is obtained when all fracture energy of the material is dissipated. A first preliminary description of the procedure used by the proposed model has been already presented in [27,28].

This work proves that the proposed model it capable of simulating material failure due to Ultra Low Cycle Fatigue by its own, without the need of any other damage variable computed independently of the plastic formulation. Besides, the proposed approach is not only capable of predicting material failure for regular and non-regular cyclic loads, but it is also capable of coupling cyclic loads with monotonic loads, which allows to predict that the structure will fail sooner if the monotonic load is applied after several hysteresis cycles, than if these cycles are not applied. This capability is obtained as a consequence of the fact that the material failure is predicted by the plastic non-linear constitutive equation itself. Another advantage of the formulation proposed is that it is capable of using any yield and potential surfaces to characterize the material, which increases its applicability to different steel alloys.