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Analysis of bonded joints with laminated adherends by a variable kinematics layerwise model

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A displacement-based zig-zag plate model with variable in-plane and through-the-thickness representation and fixed degrees of freedom is developed and applied to the analysis of bonded joints. Just the in-plane displacements and the shear rotations of the middle plane are used as primary variables. The continuity functions enable an *a priori* fulfilment of the out-of-plane stress contact conditions at the interfaces between adjacent layers. High-order contributions to displacements are included to meet the stress boundary conditions at the upper and lower faces and to allow the in-plane representation to be varied, e.g. from the adherends to the overlap. In this way, it is possible to better simulate the variation of solutions and the stress boundary conditions at the ends of the overlap. Closed form expressions of these quantities are obtained using symbolic calculus, which, once incorporated in the model, simplify the obtainment of governing equations and speed-up computations. As the representation can vary from point to point, the present model permits an accurate analysis of laminates with general boundary conditions and of bonded joints under a unified approach. Applications are presented to sample cases of bonded joints with laminated adherends using appropriate series expansions of displacements. Linear and nonlinear benchmark test cases for single- and double-lap joints taken from the literature are considered. The results show that accurate stress predictions are computed with a low computational effort in all the cases considered. A good accuracy is achieved just using one component in the series expansion, which implies solving a 3×3 system.

1. Introduction

Bonded joints offer a combination of improved performance and manufacturing advantages with respect to traditional mechanical fastening, like better vibration isolation, accommodation of thermal expansion mismatch and hygrothermal swelling, better aesthetic appearance and better aerodynamic capabilities.

In particular, adhesive bonding gives a more gradual transfer of load between the structural elements, thus achieving a more uniform stress distribution. As a direct consequence, adhesive bonding is the most suited technique for joining components when stress concentrations due to mechanical fastening have potentially catastrophic effects on strength and fatigue life, like for composite laminates as discussed by Her [1].

Simulating bonded joints is a rather complex matter that requires in many cases to account for geometric and material nonlinearity, to take into consideration the stress boundary conditions in a point form and to carry out the analysis with a sophisticate modelling of out-of-plane stress and strain fields. As shown by the comprehensive literature reviews by Kuno [2], Vinson [3] and by da Silva et al. [4], many analytical, numerical and experimental studies have been published over years.

A serious design concern of adhesively bonded joints is the evaluation of the stress and strain fields across the joint and how they are influenced by geometry, materials, loading conditions, temperature and moisture effects. A considerable effort has been put forward by many researchers to fully understand the intricacies in the behaviour of bonded joints and the degree to which the stress states and the failure modes are influenced. The analysis of bonded joints can be carried out via finite element analysis (FEA) without introducing any simplifying assumption that could limit accuracy even in the case of complex joint geometries and complex material models including nonlinear effects. However FEA requires a quite long preparation and a large computing time. To overcome this problem, analytical models (AM) based on simplifying assumptions and finite-difference schemes are extensively used for studying joints. Analytical [5–10], finite element [11–14] and finite-difference [15] solutions show that even when

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the adherends and the adhesive materials are assimilable as homogeneous and isotropic, stresses and deformations are complex due to the different elastic moduli of adhesive and adherends and the enforcement of the stress boundary conditions. Results of the analyses show non-uniform shear strain and stress distributions in the adhesive layer caused by the progressive reduction of the strain in the adherends along the overlap and the continuity of the adhesive/adherends interface. The out-of plane transverse shear and normal stresses in single- (SLJ) and double-lap (DLJ) bonded joints have a peak close to the edges of the bonding layer, which can lead to the premature failure of the joint in service. The peeling stress can be larger than the shearing stress, thus becoming the dominant effect at the edges, as shown, e.g., by Nemes and Lachaud [16]. The readers can find a comprehensive discussion and extensive assessments in the papers by da Silva et al. [4,17].

Most AM are stress-based models that still maintain some of the simplifying assumptions of the pioneering models (Volkersen [18], Goland and Reissner [19] and Hart-Smith [20]) in order to allow a fast, low cost solution in closed form. Often shear stress is assumed constant across the thickness of the adhesive, or shear and peel stresses in the adhesive layer are calculated solving a plane strain problem, because the adhesive is assumed to deform only in shear and the adherends are assumed to be rigid, though these hypotheses could be not realistic in all cases. With the advent of bonded laminated composites as primary structures in the 80's, more complex AM that consider the effects due to the deformation of the adherends and of the adhesive have been developed in order to adequately treat joints with laminated adherends. In this case, it is of primary importance to accurately account for the large variation of the transverse shear and normal stresses across the thickness that rise in consequence of the different properties of the layers constituting the joint. However, trying to keep these improved models as simple as possible, often the stress-free boundary conditions at the ends of the overlap are still disregarded, along with the bending effect due to the eccentric load path of SLJ. Moreover, the deformability of adherends is still accounted for with simplified techniques, as in [19,20]. Refined AM with improved predictive capability are reviewed by da Silva et al. [4], Gustafson et al. [5], Diaz Diaz et al. [6] and Radice and Vinson [7]. These models, depending on the complexity of their governing differential equations, can be solved either in closed or numerical form. For examples, the models by Renton and Vinson [21], Srinivas [22] and Allman [23] are cited as example that account for the transverse shear and normal deformations of adhesive and adherends, satisfy the stress-free boundary conditions ([21,22]) and consider the effects of bending, stretching and shearing in the adherends and the tearing actions in the adhesive ([23]). Bonded joints with adherends made of composite materials have been recently studied by Diaz Diaz et al. [6] using a stress based layerwise model obtained by stacking Reissner–Mindlin plates, which requires to solve a system of 29 equations in 29 unknown parameters.

Because the assumptions of AM can affect the accuracy of results, an increasing number of 3-D FEA and of finite-difference 3-D schemes results for bonded joints have been published in the last years. The papers by Khalili et al. [12] and Diaz et al. [13] dealing with 3-D FEA of SLJ with CFRP adherends and epoxy adhesive and the paper by Andruet et al. [11] dealing with geometric nonlinear effects are cited as examples. Finite-difference 3-D schemes as an alternative to FEA and AM are overviewed by Xu and Li [15].

As discussed by Adams and Mallick [24], whenever a complex representation aimed at realistically simulating the stress fields is employed, a numerical scheme should be used for solving, even in the case of AM. However, this does not result disadvantageous and unpractical compared to simpler AM allowing for closed-form solutions, because costs are still saved with respect to 3-D FEA without significant accuracy loss. Besides, AM are not affected by stress singularities at the edge interfaces like 3-D FEA.

Geometric and material nonlinear effects and adherends with dissimilar thickness were recently considered by Mortensen and Thomsen [25] and Smeltzer [26], using refined AM. Damage of joints under impact loading was considered by Vaidya et al. [27].

The classical laminated plate theory (CLPT) and the first-order shear deformation plate theory (FSDPT) were used for analysis of bonded joints with laminated composite adherends by Mortensen and Thomsen [25], Wah [28] and Yang and Pang [29]. These applications shed light on whether the analysis of bonded joints can be successfully carried out with ordinary displacement-based models which are the tools customarily employed for the analysis of laminated structures. Nevertheless the adhesive peel stress is disregarded in the constitutive equations and the transverse shear stress is neglected or assumed constant across the thickness (as a consequence, equilibrium is not satisfied at the interfaces and the stress free conditions at the end of the overlap are not met), these studies have shown that the results of displacement-based models can be in a good agreement with those of finite element models, as shown e.g. in [29]. Accordingly further research is motivated in this field because even better results could be obtained without the mentioned infringements. Moreover, displacement-based AM are of interest for analysis of bonded joints because the same approach can be used across the overlap and far from it, so designers can carry out a realistic analysis of the structures in the bonded region and outside contemporaneously and with the same tool.

As a contribution towards refinement of studies with displacement-based approaches, in this paper the analysis of bonded joints with laminated adherends is carried out assuming as structural model a zig-zag model with variable kinematics and a fixed number of functional degrees of freedom (d.o.f.). It constitutes a refined version of the layerwise model recently developed by Icardi and Sola [30] and successfully applied to the analysis of laminated and sandwich composites in Ref. [31], which is here reformulated in order to treat bonded joints through inclusion of a variable in-plane representation.

The paper is structured as follows. First, the modelling assumptions are discussed and justified, along with loading and boundary conditions requirements. Then, numerical results for benchmark test cases are presented and discussed.

2. Structural model

As a necessary premise to the discussion of the structural model, the simulation of joints is briefly overviewed at the light of the layerwise effects that rise in the overlap and in the multilayered structures constituting the adherends, along with the loading and boundary conditions that should be fulfilled.

2.1. Layerwise effects

A variety of models with a progressively refined predictive capability have been developed in the past years using different techniques, as discussed above. Bonded joints with laminated adherends need appropriate displacement-based models whose representation satisfies the kinematic and stresses boundary conditions and the local indefinite equilibrium equations at any point. Due to this latter condition, the out-of-plane stresses should be continuous at the material interfaces. As a consequence, the displacements should be represented as piecewise continuous functions having appropriate discontinuous derivatives at the layer interfaces.

These models are largely adopted for analysis of composites, while they are only seldom employed for studying joints, since the stress boundary conditions cannot be easily enforced. On the contrary, stress based and mixed models that assume the displacements separately from the stresses are widespread because they facilitate the enforcement of the boundary