International Journal of Impact Engineering 42 (2012) 48–58

Contents lists available at ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

A model for statistical variation of fracture properties in a continuum mechanics code

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ARTICLE INFO

Article history:
Received 1 December 2009
Received in revised form
22 June 2010
Accepted 28 September 2010
Available online 22 December 2010

Keywords:
Statistical fracture
Johnson–Cook fracture model
Weibull statistics
Size effect
Behind armor debris

ABSTRACT

Continuum mechanics codes modeling failure of materials historically have considered those materials to be homogeneous, with all elements of a material in the computation having the same failure properties. This is, of course, unrealistic but expedient. But as computer hardware and software has evolved, the time has come to investigate a higher level of complexity in the modeling of failure. The Johnson–Cook fracture model is widely used in such codes, so it was chosen as the basis for the current work. The CTH finite difference code is widely used to model ballistic impact and penetration, so it also was chosen for the current work.

The model proposed here does not consider individual flaws in a material, but rather varies a material's Johnson–Cook parameters from element to element to achieve inhomogeneity. A Weibull distribution of these parameters is imposed, in such a way as to include a size effect factor in the distribution function. The well-known size effect on the failure of materials must be physically represented in any statistical failure model not only for the representations of bodies in the simulation (e.g., an armor plate), but also for the computational elements, to mitigate element resolution sensitivity of the computations.

The statistical failure model was tested in simulations of a Behind Armor Debris (BAD) experiment, and found to do a much better job at predicting the size distribution of fragments than the conventional (homogeneous) failure model. The approach used here to include a size effect in the model proved to be insufficient, and including correlated statistics and/or flaw interactions may improve the model.

1. Introduction

Continuum mechanics codes using Lagrangian frames and those using Eulerian frames have achieved great success in modeling ballistic impact and penetration. One widely-used model for determining material failure in such simulations is the Johnson–Cook fracture model [1], which computes a path-dependent failure using the following relation for current failure strain:

$$\varepsilon^f = \left[D_1 + D_2 e^{D_3\sigma^*}\right][1 + D_4 \ln\dot{\varepsilon}^*][1 + D_5 T^*]. \quad (1)$$

In Eq. (1), D_1, D_2, D_3, D_4 and D_5 are material constants. σ^* is the ratio of mean stress to the von Mises equivalent stress, $\dot{\varepsilon}^*$ is the non-dimensional strain-rate, and T^* is the homologous temperature.

Historically, these continuum mechanics codes applied the Johnson–Cook fracture model in a deterministic fashion throughout the problem domain; i.e., for a particular material, the same set of material constants applies to every element or cell[1] containing that material. However, because of heterogeneity, real materials are not perfectly deterministic, but instead exhibit variations of properties, for example fracture properties, throughout the volume of material. A recent approach [2,3] applied statistical variations of fracture properties to materials in a Lagrangian frame. Here, the approach is applied to the Johnson–Cook fracture model in an Eulerian code, CTH [4], and extended to account for size effects, a step toward the ultimate goal of an element-size-invariant failure criterion for consistent predictions across a spectrum of system geometries. The goal of the present work is to illustrate the dramatic improvement in the character of results (especially irregular failure patterns) achieved when heterogeneous failure properties and scale effects are incorporated into a simulation. This should lead to the realization

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[1] The term element is usually applied in the context of a Lagrangian formulation, while the term cell usually applies to the Eulerian computational unit. In the following discussions, the term element will be used for both applications, for simplicity.

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doi:10.1016/j.ijimpeng.2010.09.007

that better experimental characterization of the failure distribution is needed, and provide motivation to obtain that data.

2. Statistical variation of the initial failure strain

Identification of constraints is important if one seeks to statistically perturb material parameters. If, for example, a Weibull perturbation is applied to a material parameter, then realizations of that parameter will vary from zero to infinity, and such variation must not violate any fundamental constraints on material parameters.

Consider a specimen at room temperature in a state of zero pressure (mean stress), being strained at a rate of 1/s. The strain at failure of such a specimen will be termed initial failure strain, ε_0^f. Under these conditions, Eq. (1) reduces to the following expression for the initial failure strain:

$$\varepsilon_0^f = D_1 + D_2. \tag{2}$$

Admissible Johnson–Cook material parameters must obey the constraint D1 + D2 > 0 to ensure that a positive failure strain is required to induce failure. Satisfaction of this constraint has been confirmed for numerous materials for which Johnson–Cook data sets are available [5]. However, some of those materials have negative values for D1. Therefore, D1 and D_2 may not be individually perturbed according to a Weibull distribution since there will exist realizations violating the constraint $D_1 + D_2 > 0$. However, their sum, the initial failure strain ε_0^f, can be perturbed via a Weibull distribution (or any other distribution for which negative realizations are impossible[2]).

Weibull [6] considered the statistics of failure events (i.e., failure of systems), for example the fatigue life of a rotating steel beam.[3] Here, the applicability of Weibull statistics at an *element level* within a shock physics code is explored under the following progression of assumptions:

The failure strain ε^f in an elemental volume of the system is strongly dependent on the criticality[4] of the flaws in that element.

The criticality of an isolated flaw is not necessarily Weibull-distributed, but a Weibull distribution is reasonable for an ensemble of flaws of random orientations if the population contains many small flaws and relatively few large flaws [7].

The failure strains of the ensemble of elements making up the system are also Weibull-distributed [7].

Since $\varepsilon^f = f(\varepsilon_0^f)$, the initial failure strain in each of the many volume elements making up the system is also Weibull-distributed. This assumption differs from the approach in [3], as discussed later.

Therefore the initial failure strain will be Weibull-distributed throughout the elements of a material.[5]

Eq. (1) does not explicitly contain the initial failure strain; to use a statistically varying initial failure strain, Eq. (1) is rearranged algebraically to the following form:

$$\varepsilon^f = \left[\varepsilon_0^f - D_2\left(1 - e^{D_3\sigma^*}\right)\right][1 + D_4 \ln\dot{\varepsilon}^*][1 + D_5 T^*]. \tag{3}$$

As mentioned, the failure strain in the standard Johnson–Cook model is deterministic. However, variations in micromorphology of a material lead to variations in failure strain. Probabilities associated with such variations lead to a dependence of failure strain on the specimen size. Larger samples are more likely to contain a critically oriented or critically large flaw, making larger samples statistically more prone to failure at a given strain. Below, a mathematical framework [7] is developed to account for these macroscale effects of variability in flaw morphology without requiring actual details about crack sizes, shapes, orientations, or clustering.

Consider a sample of volume V containing exactly one flaw. Let the sample be subjected to a prescribed strain, ε. Whether or not the sample will fail is uncertain because of uncertainties in flaw morphology such as crack orientation, size, or shape. If, for example, the strain state is tensile in one direction and compressive in another, the sample is certainly safe from failure if the crack normal is aligned with the compressive direction, but flaw orientations are unknown. Even when all principal strains are compressive, a flaw can fail under shear if it is critically oriented and sufficiently large. However, flaw size is unknown.

Regardless of the basis of uncertainty, let $g(\varepsilon)$ symbolically denote the probability that the sample is safe from failure at the applied strain ε (this single-flaw probability is not expected to be Weibull-distributed). Under a non-interaction assumption, a sample containing N flaws is safe from failure only if every flaw in the sample is safe from failure, giving the probability that the sample is safe to be $P_s = [g(\varepsilon)]^N$. This is expected to be an upper-bound since flaw interactions are expected to reduce the likelihood that the sample is safe. For elastic properties, a non-interaction assumption is valid to very high crack densities [8], but such cannot be assumed for failure properties.

Let $n = N/V$ denote the flaw density and let P_f denote the probability of failure. Thus, for the non-interaction model,

$$P_f = 1 - P_s = 1 - [g(\varepsilon)]^N = 1 - [g(\varepsilon)]^{nV}. \tag{4}$$

To allow for the effect of flaw interactions in an approximate way, suppose that an increase in flaw density causes P_f to increase in a way similar to intensifying the strain in a non-interaction model. Then the non-interaction model can be generalized to account for flaw interactions by multiplying the strain by an intensifier function $H(n)$ to give

$$P_f = 1 - \{g[\varepsilon \cdot H(n)]\}^{nV}. \tag{5}$$

The strain intensifier function $H(n)$ is expected to be a monotonically increasing function of crack density so that an increase in crack density would lead to an increase in apparent strain in a non-interaction model and, therefore, an increase in P_f.

Unfortunately, both the g and H functions, as well as the flaw density n, are unknowable from a practical perspective. As previously argued, a Weibull distribution is to be assumed for the multiple-flaw elements. The Weibull distribution function [6] is

$$P_f = 1 - \exp[-\phi(\varepsilon)], \tag{6}$$

where $\phi(\varepsilon)$ is a material function to be measured in the laboratory by repeated testing of the strain at failure. Applying the definition of a Weibull distribution, the material function in Eq. (6) would be of the form

$$\phi(\varepsilon) = \left(\frac{\varepsilon}{a}\right)^m. \tag{7}$$

[2] For this reason, a Gumbel distribution would be inappropriate since its realizations have no lower bound.

[3] Subsequently, system will refer to a component of a simulation, e.g., an armor plate or a penetrator.

[4] Criticality is the propensity (or probability) of a flaw to initiate failure of the system under a given state of stress, for example the size of the flaw might affect its criticality.

[5] To be sure, these assumptions are tenuous, as little or no data exists on which to base an understanding of the correct distribution of flaws in a material. And the distribution may well be different for different materials, e.g., ceramic versus metal. Nontheless, applying a Weibull distribution to the initial failure strain is a reasonable starting point, from which the efficacy of such a statistical modeling approach may be judged. A desirable outcome of the present work is to motivate experimenters to obtain such data.