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A modification of Shang–Wang fatigue damage parameter to account for additional hardening

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ABSTRACT

Based on the critical plane approach, the Shang–Wang fatigue damage parameter is analyzed. It is discovered that the effect of the additional cyclic hardening on fatigue life cannot be reflected by the normal strain excursion well. In order to solve this problem, this fatigue damage parameter is modified by using the Huber–Mises criterion. The modified damage parameter combined maximum shear strain range with normal strain range on the critical plane, and a new stress-correlated factor is introduced to account for the additional cyclic hardening. It is demonstrated that the modified criterion gives satisfactory results for all the six checked materials.

1. Introduction

One of the main causes of failure in engineering components and structures is fatigue failure. With the development of modern industry, the members and components of engineering structure in service are usually subjected to non-proportional cyclic loading which lead to the changing of the principal stresses and strains directions during a loading cycle, and this may results in additional hardening of the materials. The additional hardening of the materials, which is caused by the rotation of the principle stress and strain axes, is considered to have tighter relationship with the reduction of fatigue life under non-proportional loading compared with that under proportional loading [1–3]. The fatigue life of 304 stainless steel decreased 90%, while hardening increased 100% under a circular loading path. For Inconel 718 alloy, the fatigue life decreased 50%, while hardening increased 10% [3]. Similar results were also obtained by both Itoh et al. [4] and Chen et al. [5]. Therefore, the key problem in evaluating fatigue damage in these circumstances is the necessity of using multiaxial fatigue damage criteria based on aspects of the loading history.

Reviews of available multiaxial fatigue life prediction methods are presented by Fatemi and Socie [1], Varvani-Farahani [6], Lohr and Ellison [7], Brown and Miller [8], Wang and Brown [9], Shang and Wang [10], etc. Fatigue analysis using the concept of a critical plane of maximum shear strain is very effective because the critical plane concept is based on the physical observations that cracks initiate and grow on preferred planes. Because those models provide a physical interpretation of the fatigue damage process and define a critical plane on which fatigue cracks initiate and grow, critical plane approaches have been generally accepted. However, the non-proportional cyclic hardening factor should be considered in fatigue life prediction under non-proportional loading.

In the present study, the drawback of the Shang–Wang (SW) fatigue damage parameter is analyzed firstly, and then a modified multiaxial fatigue damage parameter for various proportional and non-proportional strain paths is developed. In the modified damage parameter, a stress-correlated factor is introduced to take into account the effect of the additional cyclic hardening caused by non-proportional loading.

2. Multiaxial fatigue damage model

2.1. Characteristics of the strains on the critical plane

In Fig. 1 it is schematically drawn a thin-walled tubular specimen subjected to combined tension and torsion loadings. The strain tensor for the thin-walled tubular specimen subjected to axial and torsional fatigue under strain-controlled loading conditions is given as

$$\Delta\varepsilon_{ij} = \begin{bmatrix} \Delta\varepsilon_x & 1/2\Delta\gamma_{xy} & 0 \\ 1/2\Delta\gamma_{xy} & -\nu_{eff}\Delta\varepsilon_x & 0 \\ 0 & 0 & -\nu_{eff}\Delta\varepsilon_x \end{bmatrix} \quad (1)$$

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Nomenclature

E	modulus of elasticity	$2N_f$	number of reversals to crack initiation
$R_{p0.2}$	yield stress (0.2%)	N_P	predicted life
R_u	ultimate tensile strength	N_T	experimental life
v_e	elastic Poisson's ratio	ε_n^*	normal strain excursion
v_p	plastic Poisson's ratio	$\Delta\gamma_{\max}$	maximum shear range
v_{eff}	effective Poisson's ratio	$\Delta\varepsilon_n$	normal strain range on the critical plane
σ_f'	axial fatigue strength coefficient	$\bar{E}$	mean error factor
b	axial fatigue strength exponent	$\Delta\varepsilon$	applied axial strain range
ε_f'	axial fatigue ductility coefficient	$\Delta\gamma$	applied shear strain range
c	axial fatigue ductility exponent	λ	strain ratio, $\Delta\gamma/\Delta\varepsilon$
K'	cyclic strength coefficient	n	number of test data
n'	cyclic strain hardening exponent	ϕ	phase shift

If the applied strains are sinusoidal, the normal strain and the shear strain on the maximum shear plane which makes an angle α_c with the thin-walled tubular specimen axis are given by Kanazawa et al. [11]

$$\gamma_{\max}(t) = \frac{1}{2}\Delta\varepsilon\sqrt{\left[\lambda\cos 2\alpha_c\cos\varphi - (1+v_{eff})\sin 2\alpha_c\right]^2 + \left[\lambda\cos 2\alpha_c\sin\varphi\right]^2} \times \sin(\omega t + \eta) \tag{2}$$

$$\varepsilon_n(t) = \frac{1}{4}\Delta\varepsilon\sqrt{\left[2(1+v_{eff})\cos^2\alpha_c - 2v_{eff} + \lambda\sin 2\alpha_c\cos\varphi\right]^2 + \left[\lambda\sin 2\alpha_c\sin\varphi\right]^2} \times \sin(\omega t - \xi) \tag{3}$$

where

$$\tan\xi = \frac{\lambda\sin 2\alpha_c\sin\varphi}{(1+v_{eff})\cos 2\alpha_c + (1-v_{eff}) + \lambda\sin 2\alpha_c\cos\varphi} \tag{4}$$

$$\tan\eta = \frac{-\lambda\cos 2\alpha_c\sin\varphi}{\lambda\cos 2\alpha_c\cos\varphi - (1+v_{eff})\sin 2\alpha_c} \tag{5}$$

$$\tan 4\alpha_c = \frac{2\lambda(1+v_{eff})\cos\varphi}{(1+v_{eff})^2 - \lambda^2} \tag{6}$$

$$\lambda = \Delta\gamma/\Delta\varepsilon \tag{7}$$

In Eqs. (2)–(7), ϕ is the phase shift between the tensional strain and torsion strain. v_{eff} represents the Poisson's ratio which is given by

$$v_{eff} = \frac{v_e\Delta\varepsilon_e + v_p\Delta\varepsilon_p}{\Delta\varepsilon_e + \Delta\varepsilon_p} \tag{8}$$

where v_e and v_p are the elastic and plastic Poisson's ratio, respectively. The axial elastic strain range is calculated by using Hooke's law

$$\Delta\varepsilon_e = \frac{\Delta\sigma}{E} \tag{9}$$

The plastic strain range $\Delta\varepsilon_p$ is determined by the following equation

$$\Delta\varepsilon_p = \Delta\varepsilon_x - \frac{\Delta\sigma}{E} \tag{10}$$

In Eqs. (9) and (10), $\Delta\sigma$ is the axial stress range.

The phase shift between ε_n and $\gamma_{\max}$ is $(\xi + \eta)$, range between $-\pi/2$ and $\pi/2$. From Eqs. (2) and (3), the $\Delta\gamma_{\max}$ and $\Delta\varepsilon_n$ can be obtained as follows

$$\Delta\gamma_{\max} = \Delta\varepsilon\sqrt{\left[\lambda\cos 2\alpha_c\cos\varphi - (1+v_{eff})\sin 2\alpha_c\right]^2 + \left[\lambda\cos 2\alpha_c\sin\varphi\right]^2} \tag{11}$$

$$\Delta\varepsilon_n = \frac{1}{2}\Delta\varepsilon\sqrt{\left[2(1+v_{eff})\cos^2\alpha_c - 2v_{eff} + \lambda\sin 2\alpha_c\cos\varphi\right]^2 + \left[\lambda\sin 2\alpha_c\sin\varphi\right]^2} \tag{12}$$

Ref. [12] has verified the applicability of the sinusoidal approach for the triangle wave loading. Therefore, for thin-walled tubular specimen, the sinusoidal approach is used to calculate the orientation of the critical plane and the damage parameter in this paper.

2.2. The Shang and Wang multiaxial fatigue model

Critical plane models proposed by earlier workers such as Fatemi and Socie [1], Lohr and Ellison [7], Brown and Miller [8], were based on a physical interpretation of the fatigue process whereby cracks were observed to form and grow on particular planes, known as critical plane. One of the existing multiaxial fatigue models proposed by Shang and Wang [10] combines the maximum shear range $\Delta\gamma_{\max}$ with the normal strain excursion ε_n^* between adjacent turning point as an equivalent strain by means of Huber–Mises criterion, and giving the following damage parameter (SW parameter)

$$\Delta\varepsilon_{eq}^{cr}/2 = \left[\varepsilon_n^{*2} + (\Delta\gamma_{\max}/2)^2/3\right]^{\frac{1}{2}} \tag{13}$$

where the normal strain excursion ε_n^* can be expressed by

$$\varepsilon_n^* = \frac{1}{2}\Delta\varepsilon_n[1 + \cos(\xi + \eta)] \tag{14}$$

Shang and Wang considered that this damage parameter (Eq. (13)) can be used as a unified fatigue damage criterion under both proportional and non-proportional loading. However, in the

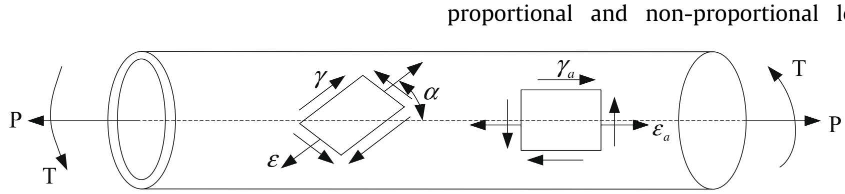

Fig. 1. Strain state of the tension–torsion specimen.