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Damage mechanics method for fatigue life prediction of Pitch-Change-Link

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ABSTRACT

According to the law of thermodynamics, a new damage evolution equation is advanced in the present study to predict the fatigue life of the Pitch-Change-Link component. As its premise, the fatigue life prediction method for smooth specimens under the repeated loading with constant strain amplitude is constructed. In addition, based on the theory of conservative integral, the closed form relation for notched specimens between the maximum stress and fatigue life is derived, by reference to which the material parameters in damage evolution equation are obtained with fatigue experiment data of standard specimens. The damage mechanics-finite element method operates with APDL language code on the ANSYS platform. Finally, the mean fatigue crack initiation life of Pitch-Change-Link is predicted with the newly proposed method, that is damage mechanics-finite element method. What's most important, the calculated results comply with the experimental data.

1. Introduction

Strength, stiffness and fatigue life are the basic requirements for engineering structure. Most engineering components are subject to cyclic load, and fatigue damage is one of the main failure modes in engineering structures. The peak value of cyclic load is far lower than static strength. So the study on the fatigue life prediction method is of great importance.

The general method of fatigue life prediction in engineering is based on statistic analysis. This method needs a lot of experimental data, and it is problematic for predicting fatigue life of components from experimental data of standard specimens [1]. Hence, it is important to formulate a method to predict the life of the components for an easier prediction of the fatigue life of the structure of components. Damage accumulation theory is such an important method for fatigue life prediction under the repeated with variable amplitude. For many years, engineering designers have used Palmgren–Miner law, linear rule [2,3] to predict fatigue life,

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \tag{1}$$

where D represents the damage extent. It is easy to predict the fatigue life by this method, so it is used widely in the industrial community.

In order to describe the damage evolution process accurately, many non-linear theories are proposed, such as bilinear damage accumulation model [4], Marco–Starkey non-linear damage accumulation model [5], Henry non-linear damage accumulation model [6], and so on. On the basis of these damage accumulation models, the components' fatigue life can be predicted through simple experiments. In addition, fracture mechanics is introduced into the analysis of fatigue problems.

Recently, a new approach based on damage mechanics of continuous medium (continuum damage mechanics) has been proposed [7–10]. This theory deals with the mechanical behavior of a deteriorated medium on the macroscopic scale. In continuum damage theories, the constitutive equations for damaged materials are usually constructed based on the concept of effective stress, the strain equivalence hypothesis or the irreversible thermodynamic theory. Damage models are concerned with isotropic damage and anisotropic damage. The isotropic damage model is characterized with simple constitutive relation and few damage parameters, so it enters into wide use in engineering. According to the irreversible thermodynamic theory, the damage evolution equation is obtained, which can be represented as follows:

$$\frac{dD}{dt} = F(Y, D) \tag{2}$$

Pitch-Change-Link is an important and familiar component in the rotating system of helicopters. The Pitch-Change-Link in this paper is the real engineer component of a in-serviced helicopter. By applying the above theory, a new non-linear damage evolution equation is constructed. First of all, in accordance with the method of variables separation and the theory of conservative integral, the damage evolution equation is integrated to express the relation between the stress and fatigue life under the repeated loading with

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constant amplitude. Secondly, the material parameters are obtained by the fatigue test results of standard specimens. Thirdly, the closed form solution is used to predict the Pitch-Change-Link's fatigue crack initiation life. Finally, damage mechanics-finite element method, which deploys the APDL language for further development on ANSYS platform, is used to predict the fatigue crack initiation life of the Pitch-Change-Link also. With the closed solution method and the numerical solution of damage mechanics-finite element method, the fatigue life thus derived accords with the experimental data.

2. Model of damage evolution

2.1. Constitutive relation and damage degree

The linear elastic constitutive relation without damage is

$$\sigma_{ij} = \delta_{ij}\lambda\delta_{kl}\varepsilon_{kl} + 2\mu\varepsilon_{ij} \tag{3}$$

where σ_{ij}, ε_{ij} stand for stress components and stain components, respectively. λ, μ are Lame Constants:

$$\lambda = \frac{Ev}{(1+v)(1-2v)}, \quad \mu = G = \frac{E}{2(1+v)} \tag{4}$$

and E is the Young's Modulus without damage, v is the Poisson ratio, G is the shear modulus.

Under the fatigue loads, the deterioration of material can be described by the reduction of the stiffness [11–14]. The concept of the damage extent is introduced to express the reduction of the stiffness, such as:

$$D = \frac{E - E_D}{E} \tag{5}$$

where E is the Young's Modulus without damage, and E_D is the Young's Modulus with damage. As E_D ranges from 0 to E, so D varies between 0 and 1. Before crack initiation, $D < 1$, damage is invisible. At instance of crack initiation $D = 1$, damage will become visible. Here, the crack is only a physical concept. In the damage mechanics-finite element method, the crack initiation is defined when the critical element damage degree is equal to 1. The size of element should be determined by convergence verification.

From Eq. (3) to Eq. (5), the constitutive relation with damage is derived as follows:

$$\sigma_{ij} = (1-D)\delta_{ij}\lambda\delta_{kl}\varepsilon_{kl} + 2(1-D)\mu\varepsilon_{ij} \tag{6}$$

It indicates the coupling relation between the damage extent and the stress component.

Under the uniaxial loading condition, the constitutive relation is:

$$\sigma = E(1-D)\varepsilon \tag{7}$$

2.2. Damage driving force

As Fatigue failure is an irreversible thermodynamics process, so according to the law of thermodynamics [15], the damage driving force can be expressed as follows:

$$Y = -\rho\frac{\partial f}{\partial D} \tag{8}$$

where ρ represents the medium mass density and f stands for the free energy per unit mass.

During the isothermal process,

$$\rho f = W = \int \sigma_{ij} d\varepsilon_{ij} \tag{9}$$

where ρf is free energy per unit volume, W is strain energy density.

From Eq. (6) to Eq. (9), the stain energy density with damage is:

$$W = \frac{1}{2}(1-D)C_{ijkl}\varepsilon_{kl}\varepsilon_{ij} \tag{10}$$

where C_{ijkl} is:

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{11}$$

When absorbing the uniaxial loading, the damage driving force is expressed as follows:

$$Y = -\frac{\partial W}{\partial D} = \frac{1}{2}E\varepsilon^2 = \frac{W}{1-D} \tag{12}$$

2.3. Damage evolution equation

According to the law of thermodynamics, under different conditions, the damage evolution equation can be established in different ways:

(1) when $Y_{\max} > Y_{th,k}$

$$\frac{dD}{dN} = \beta_k\frac{(Y_{\max}^{\frac{1}{2}} - Y_{th,k}^{\frac{1}{2}})^{m_k}}{(1-D)^{m_k}} \tag{13}$$

(2) when $Y_{\max} < Y_{th,k}$

$$\frac{dD}{dN} = 0 \tag{14}$$

where β_k, m_k are material parameters, $Y_{th,k}$ is the threshold of damage driving force. β_k, m_k and $Y_{th,k}$ are function of stress concentration factor K_T.

2.4. Fatigue life prediction for smooth specimen

From Eq. (12) to Eq. (14), when $K_T = 1$, damage evolution equation is derived as follows:

$$\frac{dD}{dN} = \alpha_1\frac{(\varepsilon_{\max} - \varepsilon_{th,1})^{m_1}}{(1-D)^{m_1}} \tag{15}$$

where $\varepsilon_{\max}$ is the maximum strain in each cycle under repeated loading, and $\varepsilon_{th,1}$ is the threshold of strain when $K_T = 1$. Comparison between Eq. (13) and Eq. (15) gives:

$$\alpha_1 = \beta_1\left(\frac{E}{2}\right)^{\frac{m_1}{2}} \tag{16}$$

From Eq. (7), we have

$$\left.\begin{aligned} \varepsilon_{\max} &= \tfrac{1}{E(1-D_{0,1})}\sigma_{0,\max} \\ \varepsilon_{th,1m} &= \tfrac{1}{E(1-D_{0,1})}\sigma_{th,1} \end{aligned}\right\} \tag{17}$$

where $D_{0,1}$ is the initial damage extent when $K_T = 1$, and $\sigma_{0,\max}$ is the maximum stress with initial damage $D_{0,1}$. $\sigma_{th,1}$ is threshold of stress when $K_T = 1$.

When we integrate Eq. (15) from $D = D_{0,1}$ to $D = 1$, and consider Eq. (15), the number of cycles to failure under a given load can be obtained:

$$\lg N_f = \lg\left[\frac{E^{m_1}}{\alpha_1(m_1+1)}(1-D_{0,1})^{2m_1+1}\right] - m_1\lg(\sigma_{0,\max} - \sigma_{th,1}) \tag{18}$$

So, the fatigue curve will be different for different value of $D_{0,1}$. If $D_{0,1m}$ is the $D_{0,1}$ corresponding to the mean fatigue curve, then, from Eq. (18), we have

$$\lg N_f = \lg\left[\frac{E^{m_1}}{\alpha_1(m_1+1)}(1-D_{0,1m})^{2m_1+1}\right] - m_1\lg(\sigma_{0,\max} - \sigma_{th,1m}) \tag{19}$$

where $\sigma_{th,1m}$ is the stress threshold when $D_{0,1} = D_{0,1m}$.