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Plastic dissipation energy at a bimaterial crack tip under cyclic loading

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ABSTRACT

A theory has been introduced that relates the fatigue crack growth rate to the total plastic energy dissipated ahead of a crack tip under cyclic loading. Due to advances in computational efficiency, the reversed plastic zone can now be resolved adequately using the finite element method. The authors have previously published results for the plastic dissipation energy for cracks in homogeneous materials under mixed mode I and II loading. The purpose of the current research is to expand that set of results to include interface cracks in a general layered material. Applications of crack growth along a bimaterial interface include soldering, layered manufacturing, thermal spray coating, welding, brazing, or any other process that deposits a material onto a substrate where often it is more energetically favorable for a crack to grow along the interface than to propagate into either of the contiguous materials. Results of the plastic dissipation energy are obtained using 2-D elastic–plastic plane strain finite element analysis of a sharp crack along a bimaterial interface. Results show the plastic dissipation energy is proportional to the square of the strain energy release rate. By normalizing the results for the plastic dissipation with respect to the loads and material properties, it can be seen that the plastic dissipation energy has the greatest dependence on the mode mix ratio, followed by elastic and plastic property mismatches, respectively. Furthermore, a definition of the mode mix ratio based on dissipated energy is presented, which provides physical motivation for the characteristic length parameter needed to quantify the mode mix along a bimaterial interface.

1. Introduction

1.1. Motivation

Applications of bonded interfaces include layered manufacturing, structural coatings, electronic packaging, fiber-reinforced composites, and other processes where a material is deposited onto a substrate. It is often energetically favorable for a crack to grow along the interface, as opposed to propagate into either of the surrounding layers. This results in mixed mode I/II cracking from either the type of the externally applied loads or a mismatch in elastic properties across the interface, or both.

Material fatigue is a primary consideration when designing, analyzing, and building systems undergoing cyclic stresses, whether these be thermal or mechanical. Recently, Klingbeil introduced a model to estimate the fatigue crack growth rate in ductile materials based on the cyclic plastic dissipation energy at the crack tip and the monotonic critical strain energy release rate [1]. The results of that research for mode I loading for homogenous ductile materials show that the estimation matches the measured fatigue crack growth rate obtained from the following equation:

$$\frac{da}{dN} = \frac{1}{\mathcal{G}_c}\frac{dW}{dN} \tag{1}$$

where $\mathcal{G}_c$ is the critical strain energy release rate and dW/dN is the total plastic dissipation energy per cycle obtained by integrating the plastic strain energy density in the reversed plastic zone ahead of the crack. To ensure dimensional consistency, the plastic dissipation is calculated per unit width. Since $\mathcal{G}_c$ is determined from monotonic tests and the total plastic dissipation energy is determined numerically, Eq. (1) provides a means of estimating the fatigue crack growth rate of ductile material systems without conducting cyclic testing, thus saving appreciable time and resources when evaluating possible material systems.

The data used by Klingbeil in Ref. [1] focused on mode I fatigue data obtained from a compact-tension C(T) specimen. It is well known that fatigue cracks orient themselves to grow in mode I conditions when it is energetically favorable. There are cases, however, where a crack is likely to grow along an interface in a mixed mode I/II condition because the toughness of the interface under the given mode of loading is less than the mode I toughness of either base material. As such, Daily and Klingbeil published trends

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in the plastic dissipation energy as a function of the mode mix ratio for homogeneous layers under sustained mixed-mode loading [2]. The results show a significant (1–2 orders of magnitude) difference between the energy dissipated in pure mode I and the energy dissipated in pure mode II.

The scope of the research reported in [2] was limited to homogeneous material systems where a mixed-mode crack is able to propagate. Such cracking may be found in welded materials, layered material systems and bonded metals. In some circumstances, the a layered system with the same elastic properties but different plastic properties may exist. Therefore, Daily and Klingbeil published results for the plastic dissipation for layered systems with mismatches in the plastic properties [3]. It was found that the plastic dissipation is governed by weaker material with the cessation of the influence of the stronger material at about 5/3 the strength of the weaker layer. In a general sense, however, the materials can be dissimilar across an interface both in elastic and plastic properties. The current research is focused on calculating the plastic dissipation for the case of both an elastic and plastic mismatch in material properties across an interface.

While the dissipated energy theory shows promise, it has some limitations. For example, the effect of crack closure or crack growth is not modeled. The constitutive models do not account for differences in response upon load reversal (i.e. the kinematic hardening model assumes similar response in tension and compression). Modeling of the micromechanisms at the crack front is lumped into the overall energy quantification. Finally, the experimental observations needed to test Eq. (1) for mixed mode are a subject of ongoing research. Therefore, the numerical results presented in this paper should be viewed as first order estimations of the plastic dissipation, which is known to have influence on the fatigue crack growth rate.

1.2. Organization

This paper continues in Section 2 by describing some background of interface fracture mechanics. The concept of mode mix is examined and the need for a characteristic length is explained. Section 2.4 describes the dissipated energy approach. Following the review of previous work, the paper explores the plastic dissipation energy for a bimaterial crack. The results for the normalized plastic dissipation are mapped out as a function of the mode mix ratio, elastic mismatch, and plastic mismatch. A discussion and conclusions follow.

2. Background

The definition of the mode of loading at a crack tip is straightforward for a homogeneous material and can be found in most elementary texts on fracture mechanics. However, if a crack is growing along an interface between layers with a mismatch in elastic properties, the mode of loading becomes more difficult to describe in an unambiguous fashion.

Consider a general layered specimen as shown in Fig. 1. The layered system has two layers of isotropic materials of potentially different thicknesses. A traction free crack exists along the interface and the crack length is sufficiently long for steady-state crack growth. The loads shown are the result of a superposition analysis to impose loads in a self-equilibrating manner. The details of the superposition argument that lead to the loading shown in Fig. 1 were presented by Suo and Hutchinson in [4]. In this paper, the general layered system geometry is simplified to have equal layers and no axial loads as shown in Fig. 2.

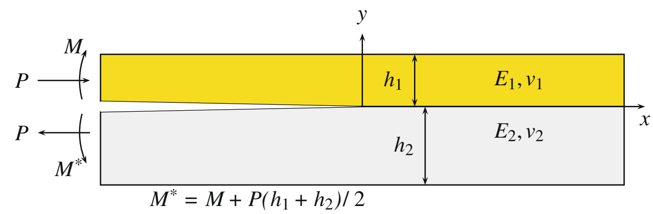

Fig. 1. General self-equilibrating, layered system with isotropic layers.

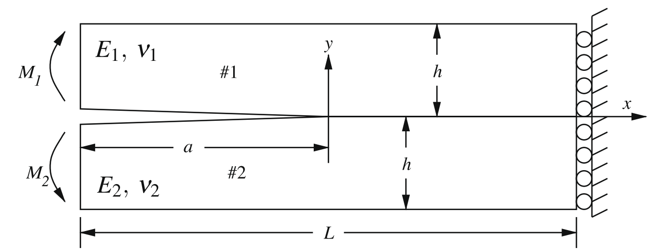

Fig. 2. Specimen geometry for mixed-mode cracking with matching layer thicknesses. Each layer is isotropic and can have different elastic and plastic properties.

2.1. Stress intensity factors

Williams [5] is credited with developing an eigenfunction expansion solution to the crack analysis problem that shows both opening (mode I) and shearing (mode II) behaviors are present at an interface crack, even when the externally applied loading is in mode I. This results from the oscillating stress singularity near the crack tip. The stress fields oscillate in a logarithmic fashion, which means the stresses change sign an infinite number of times as the distance from the crack tip approaches zero.

Further advances of the solution of the field equations of elasticity based on Mushkelishvili's complex potential technique were performed by Cherepanov [6], England [7], Erdogan [8], and Rice and Sih [9] in which a complex stress intensity factor

$$K = K_1 + iK_2 \tag{2}$$

was introduced to characterize the stress fields. The magnitude of the complex stress intensity factor is determined from the relationship:

$$|K|^2 = K_1^2 + K_2^2 \tag{3}$$

The complex stress field around the crack tip has the form

$$\sigma_y + i\sigma_x|_{y=0} = \frac{(K_1 + iK_2)r^{i\varepsilon}}{\sqrt{2\pi r}} \tag{4}$$

where ε is known as the oscillation index. The mode mix ratio of the interface stress intensity factors can be defined as

$$\phi = \tan^{-1}\frac{K_2}{K_1} \tag{5}$$

However, a more general definition of the mode mix ψ was explained by Rice [10], where he gives details about the issue of interpreting the mode mix in the context of a complex stress intensity factor. For the mode mix to be dimensionally consistent, the mode mix at a location $x = l$ is determined as

$$\psi = \tan^{-1}\frac{Im[Kl^{i\varepsilon}]}{Re[Kl^{i\varepsilon}]} \tag{6}$$

where the quantity l is an *arbitrary* characteristic length. Note that $\phi = \psi$ when $l = 1$ or $\varepsilon = 0$. Hutchinson and Suo [4,11] have champi-