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On the inverse power laws for accelerated random fatigue testing

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Abstract

This paper addresses the usage of inverse power laws in accelerated fatigue testing under wide-band Gaussian random loading. The aim is not at predicting an absolute value of fatigue life but assessing the fatigue damage relative accumulation. The widely accepted inverse power scaling laws in fatigue damage assessment is discussed, reviewing the engineering standards and pointing out their inherent limitations. A physically consistent general scaling law is obtained by rigorous mathematical analysis in the framework of random vibration theory and the rules of safe-life fatigue analysis. Simplifications of the general scaling rule are presented, highlighting conditions under which the current standard practice could provide a correct an acceptable estimation of the relative fatigue damage accumulation.

Keywords: Fatigue; Inverse power law; Random loading; Cycle counting; Accelerated testing

1. Introduction

Several methodologies have been developed for predicting fatigue life of engineering components under random vibration loads. These methods may be divided into two main groups depending on the parameters and data used in the analysis. The time-domain based analysis, which depends mainly on the stress or strain amplitudes in the time history, has been well-established (e.g. [1–3]) and widely accepted. In a time-domain approach, a cycle counting procedure and, e.g. the rainflow algorithm [4,5], a cumulative damage rule are employed. The only difference with the non-vibration applications is that random loading causes stress ranges of all magnitudes from zero to a set maximum, which must be specified via probability density distributions and the damage summation must be evaluated as an integral. However computer simulation of all possible stress-time history samples is very time consuming and at the design stages the full stress history may not be available.

On the other front, the frequency domain analyses are now an accepted practice [6–10]. These methodologies are based on expressing the fatigue life as a function of the spectral characteristics of the excitation. However, in spite of considerable efforts, no general analytical solution is yet available; damage accumulation in frequency domain is currently described by approximated laws. Most of the existing techniques are limited to the stationary Gaussian loading histories and there is no general agreement on how the effects of multi-axial loading on fatigue life can be accounted for.

Apart from random loading effects, many other random factors also influence fatigue damage accumulation, e.g. the randomness of material properties and randomness of defect distribution in components. It is recognized that the calculation of fatigue damage accumulation in absolute terms, i.e. life cycles, is still extremely difficult due to the randomness of the above factors. On the other hand, even for the simple constant or variable amplitude loading conditions, large scatters exist in fatigue life prediction. This is mainly due to the inherent limitation of the linear cumulative damage rule, i.e. the Miner's law, which is also widely employed in vibration fatigue analysis. Numerous statistical studies and fatigue tests have been

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Nomenclature

T_r	fatigue life in the operative environment
T_s	fatigue life in the testing condition
g	acceleration root mean square value (RMS)
g_r	acceleration RMS in the operative environment
g_s	acceleration RMS in the testing environment
$\bar{g}^* = \bar{g}/g$	normalized mean value of the acceleration
$\hat{g}^* = \hat{g}/g$	normalized acceleration amplitude
α	inverse power law exponent
v	frequency
$S_r(v)$	power spectral density (PSD) of the operative acceleration
$S_s(v)$	PSD of the testing acceleration
$S_{x,y}(v)$	cross PSD associated with the time dependent variables x and y
$\lambda_{(r),p}$	pth order spectral moment of the operative PSD
$N_{(r),0}$	number of zero-crossings with positive slope per unit time for the operative acceleration history
$N_{(r),p}$	number of peaks per unit time for the operative acceleration history
$\widetilde{N}_0$	total number of zero-crossings with positive slope
$\Sigma_{i,j}^{\mathrm{RF}}$	rainflow matrix
$\mathbf{M}$	structural mass matrix
$\mathbf{C}$	structural damping matrix
$\mathbf{K}$	structural stiffness matrix
$\mathbf{S}$	matrix for expressing the von Mises stress in spectral form
$F(t)$	vector of external excitation forces
$H_s(v)$	harmonic response associated to the sth mode
ϵ	vector representing the six independent component of the strain tensor
σ	vector representing the six independent component of the stress tensor
a	external acceleration vector
σ_{eq}	von Mises equivalent stress
$\tilde{\sigma}$	RMS of the von Mises equivalent stress
b	Basquin's fatigue exponent

conducted to find the statistical distribution of the damage [11].

This paper is not about proposing another method for predicting fatigue life. Rather, it aims at providing further insight into the inverse power scaling law models, which allow estimating the relative damage accumulation based on laboratory tests. It is customary to perform accelerated random vibration testing in a laboratory environment, which in terms of loading is considerably more severe than the operative one; the operative life duration is then estimated by relating the structural fatigue life tested in the laboratory condition by a proper scaling factor. The scaling law depends on the ratio of the load severities, e.g. using the root mean square values, of the two environments and also on some empirical or semi-empirical constants, which are determined from either the previous design or usage experience. A widely employed inverse power law in the aerospace industry to correlate the laboratory tests to the real service life is as follows [12–14]:

$$\frac{T_s}{T_r} = \left(\frac{g_r}{g_s}\right)^{\alpha} \tag{1}$$

where T_r is the fatigue life in terms of time duration in an actual operative environment, T_s the fatigue life corresponding to accelerated laboratory test, g_r and g_s are the root mean square (RMS) values of the actual and lab simulated accelerations (or loads), respectively, applied to the structures. The exponent α represents the slope of the material S–N curve in log–log coordinates. The inverse power scaling law in Eq. (1) was derived from the Coffin–Manson's model of fatigue life [15,16]. It is worth stressing that Eq. (1) is implemented in the MIL Standard 810 [17], which specifies the lab testing environment for the certification of military aerospace sub-systems in the US; this standard is also commonly adopted by countries belonging to the NATO. Eq. (1) simply states that the testing time in the laboratory environment is inversely proportional to the applied force RMS via exponent α. Although it has been recognised that the exponent α is the slope of the material S–N curve, the assumptions made to derive Eq. (1) are extremely reductive for the following reasons:

- Coffin–Manson's model [15] is strictly valid for a uniaxial applied force, i.e. a simple tensile/compressive stress, while Eq. (1) is applied for the safe-life estimation of complex structures, for which the operative loading and the resultant stress field may be multi-axial;
- complex structures may be made of several different materials, each having a characteristic sensitivity to fatigue damage accumulation;
- there is no rigorous proof that the acceleration RMS are sufficient to fully describe the random vibration environment, especially for wide-band excitation spectra;
- under random loading the material S–N curve exponent is generally dependent on the spectral properties of the excitation [18,19]; standard fatigue tests on materials are performed with sinusoidal excitation, but the slope of the S–N curve for an assigned material changes under random loading. The methodology developed in this paper is based on considering the S–N curve under random loading condition.

Therefore, in the light of the complexity of the phenomenon under investigation, the very simple scaling law in Eq.