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The effect of loaded volume and stress gradient on the fatigue limit

S. Norberg [a,b], M. Olsson [b,*]

[a] Scania CV AB, SE-151 87 Södertälje, Sweden

[b] Solid Mechanics, Royal Institute of Technology (KTH), SE-100 44 Stockholm, Sweden

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Abstract

In this paper an investigation of multiaxial stress based criteria and evaluation methods is presented. The criteria are used with the point, gradient and volume methods. The purpose is to determine the combination of criteria and methods that is best suited for design against the fatigue limit. The evaluation is based on elastic FE-analysis of 15 geometries for which the fatigue limit loads are known. The point method is based on the maximum values of the fatigue stress in each specimen. With the gradient method, the fatigue stress is adjusted with the relative or absolute gradient of the fatigue stress itself. With the volume method, a statistical size effect is considered, by use of a weakest link integral. Thus, the probability of fatigue depends on the fatigue stress distribution. Also, the gradient and volume methods are combined. The results show that the point and gradient methods are not good for prediction of the fatigue limit. It is recommended to use the volume method in fatigue design. It is accurate enough for prediction of the fatigue limit, straightforward to use and easy to interpret. The choice of method is much more important than the choice of criteria.

Keywords: High cycle fatigue; Weakest link; Statistical size effect; Gradient effect

1. Introduction

1.1. The importance of fatigue assessment

A machine component must meet several requirements regarding mechanical properties, such as static strength, fatigue strength, and stiffness. It must also be economical in the sense that the total cost of the whole structure, including both production and use, should be low. The amount of raw material used and the final volume of the component are thus important. In a vehicle application it directly affects the weight that needs to be transported. Unnecessary weight decreases the possible payload as well as increases the fuel consumption. Another, often more important, factor to consider is the reliability of the component or the whole structure. The reliability can be influenced by many factors. However, fatigue is usually the most important life-limiting phenomenon.

The designer must balance reliability against total cost. The goal is to achieve an optimum design that incorporates all the requirements. The industrial need for accurate fatigue predictions is obvious. Erroneous, or neglected, fatigue assessment during component design can lead to malfunction. The cost that eventually follows from product failure can be considerable for the producer as well as the consumer. Failures can also lead to accidents.

In this paper, the combined influence from fatigue assessment methods and criteria based on elastic FE-analyses of the loaded structures is investigated. The goal of the investigation is to identify criteria and methods that are good for prediction of the fatigue limit loads for a number of different geometries. In addition, the methods should be quick and easy to use.

1.2. Current methods of fatigue assessment

In [1] different criteria and the fatigue post-processor Femfat were evaluated. It was found that all the evaluated criteria were unsatisfactory. It was observed that the

* Corresponding author. Tel.: +46 8 790 75 41; fax: +46 8 411 24 18.
E-mail address: mart@hallf.kth.se (M. Olsson).

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estimations of the fatigue limit load of the different specimens are related to how severe the stress concentration in each specimen is. This, in conjunction with a difficulty to separate the criteria, lead to the conclusion that it may be more important to take the spatial distribution of the stress into account than to choose among existing criteria. The investigation presented here attempts to clarify whether the large differences in the maximum stresses of the different specimens, when evaluated at their fatigue limit loads, can be explained with the use of either the statistical size effect or the gradient effect.

Both these effects have been suggested by numerous authors [2–7]. Both effects are motivated by the fact that all empirical experience shows that the higher the maximum stress at the fatigue limit load the smaller the volume subjected to high stresses. A small volume of high stress with moderate stresses around requires a large stress gradient. This makes it reasonable that the stress gradient should be related to the allowable value of the maximum stress. If the distribution of flaws is roughly uniform and the volume subjected to a high stress is small, the worst flaw will probably be smaller than the worst flaw in a larger volume. A smaller volume can thereby probably tolerate a higher stress. The weakest link method is derived from such statistical reasoning.

1.3. Point method

The point method only requires the time history of the stress tensor to be known at the point. Information about the stress in the surroundings or any gradients or the size of the specimen is not used. The criteria used in this work are Crossland [8], Dang Van [9], Findley et al. [10], Matake [11], Papadopoulos [12], and Sines [13]. They are evaluated at all material points as described by Eqs. (1)–(8). In the equations, symbols not explicitly defined should be self explanatory. All criteria are expressed with the use of two equations. The equation to the left defines a local fatigue stress; the equation to the right compares this with a critical material value. If this critical value is exceeded at any point in the loaded specimen or component, fatigue is predicted to occur.

The Crossland criterion uses a von Mises stress modified with the maximum hydrostatic stress during the load cycle,

$$\sigma_{\mathrm{vM,a}} + \alpha_{\mathrm{C}}\sigma_{\mathrm{h,max}} = \beta_{\mathrm{C}}, \quad \beta_{\mathrm{C}} \leqslant \beta_{\mathrm{C}}^{\mathrm{crit}}. \tag{1}$$

The subscript "a" means amplitude, "vM" von Mises and "h" hydrostatic. All α- and β^{crit}-parameters are criteria specific material properties and the local value of the β-parameter is the fatigue stress of each criterion, respectively. The fatigue stress of criterion "X" is denoted β_X, thus, β_{C} is the Crossland fatigue stress, and the α- and β^{crit}-parameters are named analogously.

The Dang Van criterion is based on the use of a modified stress state with stresses $\tilde{\sigma}_{ij}(t)$. This is used to compute the maximum shear stress and the hydrostatic tension as functions of time and they are combined in the criterion,

$$\max_t(\tilde{\tau}_{\max}(t) + \alpha_{\mathrm{DV}}\sigma_{\mathrm{h}}(t)) = \beta_{\mathrm{DV}}, \quad \beta_{\mathrm{DV}} \leqslant \beta_{\mathrm{DV}}^{\mathrm{crit}}. \tag{2}$$

The modified stress state is defined by using a yield surface. A material that exhibit an infinitely low mixed hardening is assumed. This hardening will lead to the smallest possible yield surface that contains the time history of the stress tensor. The deviatoric part of the midpoint of this yield surface is subtracted from the original stress state. This leads to a stress state from which all mean shear stresses have been removed. Further, the differences between the means of the principal stresses are removed but the mean hydrostatic stress is preserved. Also, all varying parts of the original stress state are retained since the stress state is modified with a constant. In the criterion, the instantaneous values of the largest shear stress of the modified stress state, $\tilde{\tau}_{\max}(t) = (\tilde{\sigma}_1(t) - \tilde{\sigma}_3(t))/2$, and the hydrostatic stress are combined and the fatigue measure is defined as the largest value of this combination during the load cycle.

The Findley criterion is a critical plane criterion and it is evaluated on the plane on which the expression in the parenthesis in Eq. (3) attains its highest value. The fatigue stress of the criterion equals this maximum value,

$$\max_{\text{all planes}}(\tau_{\mathrm{a}} + \alpha_{\mathrm{F}}\sigma_{n,\max}) = \beta_{\mathrm{F}}, \quad \beta_{\mathrm{F}} \leqslant \beta_{\mathrm{F}}^{\mathrm{crit}}. \tag{3}$$

Here, τ_{a} is the shear stress amplitude of a plane. The shear stress amplitude is defined by the smallest circumscribed circle method [14], and $\sigma_{n,\max}$ is the largest normal stress that acts on the plane during the load cycle.

The Matake criterion is a critical plane criterion, like the Findley criterion, but it uses one measure to identify the critical plane and another to evaluate the loading of it,

$$\max_{\text{all planes}}(\tau_{\mathrm{a}}) + \alpha_{\mathrm{M}}\sigma_{n,\max}^{*} = \beta_{\mathrm{M}}, \quad \beta_{\mathrm{M}} \leqslant \beta_{\mathrm{M}}^{\mathrm{crit}}. \tag{4}$$

The normal stress $\sigma_{n,\max}^{*}$ is evaluated on the plane with the largest shear stress amplitude. It is defined as the largest normal stress that occurs on that plane during the load cycle.

The Papadopolus criterion is a critical plane criterion which uses a special type of shear stress,

$$T_{\mathrm{a,\,max}} + \alpha_{\mathrm{P}}\sigma_{\mathrm{h,\,max}} = \beta_{\mathrm{P}}, \quad \beta_{\mathrm{P}} \leqslant \beta_{\mathrm{P}}^{\mathrm{crit}}, \tag{5}$$

$$T_{\mathrm{a,\,max}} = \max_{\text{all planes}}\sqrt{\frac{1}{\pi}\int_0^{2\pi}\tau_{\mathrm{a}}^2(\chi)\,\mathrm{d}\chi}. \tag{6}$$

Here, an effective shear stress amplitude, $T_{\mathrm{a,\,max}}$ is used. It is defined as an RMS-value of a shear stress amplitude, $\tau_{\mathrm{a}}(\chi)$. On a cutting plane the stress vector, i.e. the traction vector, can be divided into a shear stress vector and a normal stress. The time history of the shear stress vector forms a trajectory on the cutting plane. This trajectory is projected onto a line in the cutting plane, that line is oriented with the angle χ. The shear stress amplitude, $\tau_{\mathrm{a}}(\chi)$, is defined as half the length of this projection, see Fig. 1.

The Sines criterion is similar to the Crossland criterion but it uses the mean hydrostatic stress instead of the maximum,