International Journal of Adhesion & Adhesives 29 (2009) 495–502

Contents lists available at ScienceDirect

International Journal of Adhesion & Adhesives

journal homepage: www.elsevier.com/locate/ijadhadh

Three-parameter, elastic foundation model for analysis of adhesively bonded joints

Jialai Wang *, Chao Zhang

Department of Civil, Construction, and Environmental Engineering, The University of Alabama, Tuscaloosa, AL 35487–0205, USA

ARTICLE INFO

Article history:
Accepted 25 October 2008
Available online 21 November 2008

Keywords:
Interface stress
Interface debonding
Adhesively bonded joint
Single-lap joint
Stiffened joint
Three-parameter elastic foundation model

ABSTRACT

A novel three-parameter, elastic foundation model is proposed in this study to analyze interface stresses of adhesively bonded joints. The classical two-parameter, elastic foundation model of adhesive joints models the adhesive layer as a layer of normal and a layer of shear springs. This model does not satisfy the zero-shear-stress boundary conditions at the free edges of the adhesive layer due to the inherent flaw of the two-parameter, elastic foundation model, which violates the equilibrium condition of the adhesive layer. To eliminate this flaw, this study models the adhesive layer as two normal spring layers interconnected by a shear layer. This new three-parameter, elastic foundation model allows the peel stresses along the two adherend/adhesive interfaces of the joint to be different, and therefore, satisfies the equilibrium condition of the adhesive layer. This model regains the missing “degree of freedom” in the two-parameter, elastic foundation model of the adhesive layer by introducing the transverse displacement of the adhesive layer as a new independent parameter. Explicit closed-form expressions of interface stresses and beam forces are obtained. The new model not only satisfies all boundary conditions, but also predicts correctly which interface has the strongest stress concentration. The new model is verified by continuum models existing in the literature and finite element analysis. The new three-parameter, elastic foundation model provides an effective and efficient tool for analysis and design of general adhesive joints.

1. Introduction

Adhesively bonded joints are widely used in composite structures to connect components due to their many advantages compared with other joining methods. However, premature failure due to debonding and peeling of the joint is the major concern of this technique. To address this concern, numerous theoretical and experimental studies have been conducted to evaluate the strength of the adhesive joint. Goland and Reissner [1] modeled (G–R model) the adhesive layer as continuously distributed shear and vertical springs. In this model, no interactions are assumed between the shear and vertical springs, and therefore, the adhesive layer is modeled as a two-parameter, elastic foundation. Simple explicit closed-form expressions of interface stresses and beam forces can be obtained by this model as demonstrated by many researchers [2,3]. The interface stresses predicted by the two-parameter, elastic foundation model reach good agreements with those obtained through continuum analysis such as finite element analysis (FEA) [4] except in a small zone at the vicinity of the edge of the adhesive layer.

To predict more accurate stress distribution of the adhesive joint, many refined models have been developed by modifying the two-parameter, elastic foundation model of the adhesive layer used in G–R model [5–17]. Martensen and Thomsen [18,19] considered the nonlinearity of the adhesive layer. Carpenter [20] used the solution based on finite element analysis as baseline to evaluate different lap-shear joint theories. The major drawback of the G–R model and its descendents mentioned above is that they do not satisfy the zero shear stress at the free edges of the adhesive layer [17]. As illustrated in [2,3], the governing differential equation of the two-parameter model is of the sixth order, which requires six boundary conditions; while there are eight boundary conditions available, including six forces and two shear stress boundary conditions. In the two-parameter, elastic foundation model, the zero shear stress boundary conditions are ignored. As a result, it predicts a maximum shear stress at the free edge of the adhesive layer. To overcome this drawback, some researchers modeled the adhesive layer as two-dimensional continuum medium [21–24]. However, these methods require complicated methods such as employing the variational principle of complementary energy or introducing higher order beam theory. This makes it difficult to use them in analysis and design [4].

In this study, we present a novel, three-parameter, elastic foundation model of adhesive joints. The model is a direct

* Corresponding author. Tel.: +1 205 348 6786; fax: +1 205 348 0783.
E-mail address: jwang@eng.ua.edu (J. Wang).

0143-7496/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijadhadh.2008.10.002

extension of the classical two-parameter, elastic foundation model in that it avoids a fundamental flaw regarding the adhesive layer. In the two-parameter, elastic foundation model, the force equilibrium conditions of the adhesive layer are not satisfied, because the peel stresses along the two adherend/adhesive interfaces of the adhesive joint are assumed to be equal to each other. In the new model, we remove this restriction on the peel stresses and assume that they are different. In this way, there are three different interface stresses existing in the adhesive layer. They are the shear stress within the adhesive layer, the peel stress along the top adherend–adhesive (TA) interface, and peel stress along the bottom adhesive/adherend (AB) interface. It will be shown later in this study that the different peel stress distributions along two interfaces are required by the equilibrium condition of the adhesive layer. To represent these stresses properly, the adhesive layer is modeled as two linear-normal spring layers interconnected by a shear layer, instead of only *one* linear-normal spring layer and one shear spring layer as used in the two-parameter, elastic foundation model. By considering the equilibrium condition of the adhesive layer, a governing differential equation of eighth order for a general configured adhesive joint is obtained in term of the axial force of the adherend. Explicit closed-form expressions of interface peel stresses along two interfaces and the shear stress through the thickness of the adhesive layer are obtained from this new model, and all eight boundary conditions are satisfied. To verify the new model, FEA results are used as reference standard because they are able to predict the location of failure initiation in the adhesive joint. Excellent agreements have been achieved by the new model and FEA on predicting interfacial stresses of two typical adhesively bonded joints. Compared with two-parameter, elastic foundation model, the present model is more accurate in predicting interfacial peel stress distribution near the edge of the adhesive layer, which is critical to evaluating the potential of debonding and predicting where the debonding can initiate. The formulation of this study is in similar fashion to the two-parameter, elastic foundation model [2,3], and the solutions are in explicit closed forms. It can be followed and implemented conveniently by other researchers.

2. Three-parameter, elastic foundation model of a general lap joint

2.1. Adhesively bonded bi-layered beam system

Consider a typical adhesive joint in which two adherends are connected through a thin layer of adhesive in the overlap segment. These two adherends are modeled as Timoshenko's beams [25] with thickness h_1 and h_2, to account for the shear deformation of the adherends. Here h_1 and h_2 are not necessarily equal to account for both the symmetric and asymmetric adhesive joints.

In this study, the analysis is focused on the overlap area of the joint. Consider a typical infinitesimal, isolated body (Fig. 1) of the overlap, which is a bi-layered beam system. The deformation of the two adherends can be written as

$$U_i(x,z_i) = u_i(x) + z_i\phi_i(x), \quad W_i(x,z_i) = w_i(x_i) \tag{1}$$

where $u_i(x)$, $w_i(x)$ and $\phi_i(x)$ $(i = 1, 2)$ are the axial, transverse displacements, and rotation of the neutral axis of adherend i, respectively; $U_i(x,z_i)$ and $W_i(x,z_i)$, $(i = 1, 2)$ are the axial and transverse displacements of adherend i, respectively; subscript $i = 1, 2$, represent the adherend 1 (top adherend) and 2 (bottom adherend) in Fig. 1, respectively; x and z_i are the local coordinates of adherend i with x-axis along the neutral axis of the beam i.

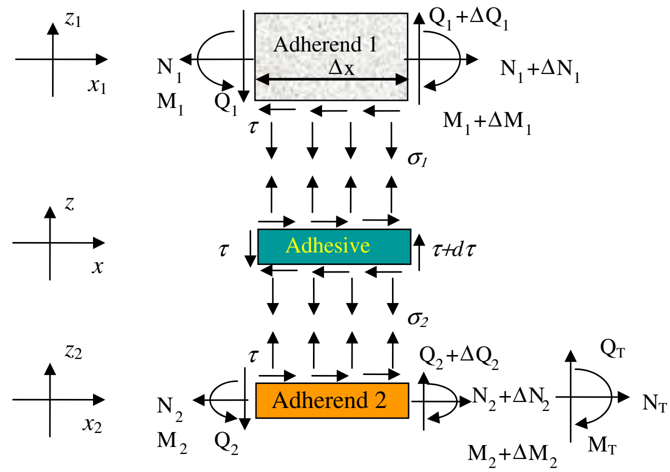

Fig. 1. Free body diagram of the adhesive joint.

By making use of the constitutive equations of individual layers, we can relate beam forces and displacements of adherends:

$$C_1\frac{du_1(x)}{dx} = N_1(x), \quad C_2\frac{du_2(x)}{dx} = N_2(x) \tag{2a}$$

$$\frac{dw_1(x)}{dx} + \phi_1(x) = \frac{Q_1(x)}{B_1}, \quad \frac{dw_2(x)}{dx} + \phi_2(x) = \frac{Q_2(x)}{B_2} \tag{2b}$$

$$D_1\frac{d\phi_1(x)}{dx} = M_1(x), \quad D_2\frac{d\phi_2(x)}{dx} = M_2(x) \tag{2c}$$

where $N_1(x)$ and $N_2(x)$, $Q_1(x)$ and $Q_2(x)$, and $M_1(x)$ and $M_2(x)$ are the internal axial forces transverse shear forces, and bending moments in adherend 1 and adherend 2, respectively; C_i, B_i and D_i $(i = 1, 2)$ are the axial, shear and bending stiffness, respectively, and they are expressed as $C_i = E_ib_ih_i$, $B_i = 5/6(G_ib_ih_i)$, $D_i = E_ib_ih_i^3/12$, where E_i and G_i $(i = 1, 2)$ are the longitudinal Young's modulus and shear modulus of beam i, respectively; b_i is the width of beam i.

Assuming that the shear stress is constant through the thickness of the adhesive layer, we can establish the following equilibrium equations by using free body diagram shown in Fig. 1:

$$\frac{dN_1(x)}{dx} = b_2\tau(x), \quad \frac{dN_2(x)}{dx} = -b_2\tau(x) \tag{3a}$$

$$\frac{dQ_1(x)}{dx} = b_2\sigma_1(x), \quad \frac{dQ_2(x)}{dx} = -b_2\sigma_2(x) \tag{3b}$$

$$\frac{dM_1(x)}{dx} = Q_1(x) - \frac{h_1}{2}b_2\tau(x), \quad \frac{dM_2(x)}{dx} = Q_2(x) - \frac{h_2}{2}b_2\tau(x) \tag{3c}$$

where $\sigma_1(x)$, $\sigma_2(x)$ are the peel stresses along the TA interface and the AB interface, respectively; $\tau(x)$ is the shear stresses in the adhesive. Note that the overall equilibrium condition requires (Fig. 1)

$$N_1(x) + N_2(x) = N_T \tag{4a}$$

$$Q_1(x) + Q_2(x) + Q_a(x) = Q_T \tag{4b}$$

$$M_1(x) + M_2(x) + N_1(x)\frac{h_1 + h_2 + h_0}{2} = M_T \tag{4c}$$

where N_T, Q_T and M_T are the corresponding resulting forces with respect to the neutral axis of adherend 2; $Q_a(x)$ is the shear force of the adhesive layer, which is given by $\tau(x)b_2h_0$; h_0 is the thickness of the adhesive layer.