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Analysis of revised fatigue life calculation algorithm under proportional and non-proportional loading with constant amplitude

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ABSTRACT

This paper reports the results of a study into a revised algorithm applied to the fatigue life calculation under cyclic multiaxial proportional and non-proportional loading. In contrast to the classical algorithm, which assumes that material parameters in the fatigue criterion are constant, the revised algorithm relates the values of these parameters to the number of cycles to failure. The objectives in this paper include verification of the convergence for the revised algorithm and evaluation of the fatigue life calculation by use of the revised algorithm. Detailed implementations of the multiaxial fatigue criteria by Matake and Papadopoulos are presented for the algorithm. The calculated fatigue lives are compared to the experimental ones for three steel grades: SAE1045, S355J2G and SM45C subjected to cyclic uniaxial and multiaxial proportional and non-proportional loading. It is shown that the revised algorithm provides a sole solution under the proportional and non-proportional loading by application of the Matake and Papadopoulos criteria. A higher convergence between experimental and calculated fatigue life is obtained for the revised algorithm than for the classical one.

1. Introduction

Fatigue phenomena associated with metallic materials have been studied since the 19th century. Since then a number of studies have been performed and many concepts proposed [1,2]. Despite the considerable progress in understanding the complexity of fatigue phenomena, new models are still being developed. Complexity of fatigue phenomena in metallic materials stem from microstructure inhomogeneity and diversity. The majority of concepts aimed at avoidance of the microstructure complexity are of phenomenological type. One of them is the critical plane approach developed on the basis of works in [3–5]. This idea is based on assumption that the fatigue crack initiation is a result of linear or nonlinear function of stress components specific to one plane – called the critical one. The critical plane orientation must coincide with the plane of crack initiation [4,6–10]. Various multiaxial fatigue criteria were proposed and verified by application of this concept [11–24]. In general, the multiaxial stress-based fatigue criterion defined by using the critical plane concept could be expressed as follows

$$f(\sigma_{ij}(t), n_i, s_i, \mathbf{K}) \leqslant \tau_{af}, \tag{1}$$

where f is the reducing function of multiaxial stress state with tensor components $\sigma_{ij}(t)$ varying in time t to the equivalent stress – comparable with uniaxial stress state τ_{af} (i.e. fatigue limit); n_i and s_i are components of the normal and tangent vectors to the critical plane, respectively; $\mathbf{K}$ represents a set of material constants. The material constants $\mathbf{K}$ are derived on the basis of experimental data, mostly from the fatigue limits under uniaxial push–pull or bending – σ_{af} and uniaxial torsion – τ_{af}. Thus, $\mathbf{K}$ is a function of τ_{af} and σ_{af} that fulfils two conditions

$$f(\tau_{af}, \mathbf{K}) = \tau_{af} \quad \text{and} \quad f(\sigma_{af}, \mathbf{K}) = \tau_{af} \tag{2}$$

The reducing function f is often used to calculate the equivalent stress amplitude $\sigma_{eq,a}$ and the result is compared with the stress amplitude τ_f from the fatigue characteristic τ_f ($\mathbf{B}$, N_f)

$$\sigma_{eq,a} = f(\sigma_{ij}(t), n_i, s_i, \mathbf{K}) = \tau_f(\mathbf{B}, N_f), \tag{3}$$

where $\mathbf{B}$ is set of regression parameters and N_f is the number of cycles to failure. The N_f value estimated based on the fatigue characteristic is called the calculated number of cycles, $N_{cal} = N_f$. However, the comparison presented in Eq. (3) could be incorrect because the $\mathbf{K}$ constants were derived by application of the fatigue limits (see Appendix A – an example using the Matake criterion). The $\mathbf{K}$ constants in Eq. (3) should ensure that calculated equivalent stress amplitudes $\sigma_{eq,a}$ under uniaxial bending σ_f and uniaxial torsion τ_f are correctly transformed to amplitudes of uniaxial torsion

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Nomenclature

A_σ, m_σ	parameters of a linear regression for fatigue characteristic under cyclic bending
A_τ, m_τ	parameters of a linear regression for fatigue characteristic under cyclic torsion
Er	residual of aim function
N	number of cycles
K	set of material constants
n	unit vector normal to the analysed plane orientation
s	unit vector pointing the analysed shear direction perpendicular to **n**
σ	stress
τ	shear stress
T(0.95)	scatter band determined from N_{exp}–N_{cal} plot which includes 95% of points
$T_{0.95}$	scatter band determined from uniaxial loading which includes 95% of experimental data
t	time

Subscripts

af	fatigue limit
cal	calculated
a	amplitude
eq	equivalent
exp	experimental
f	derived from fatigue characteristic
h	hydrostatic
m	mean value
max	maximum value
n	normal (in direction of **n**)
ns	shear (on the plane with normal **n** in **s** direction)

τ_f for an arbitrary number of cycles N_f. Thus, the **K** constants need to fulfil two conditions, which are similar to condition identified in Eq. (2). But instead of the fatigue limits τ_{af} and σ_{af}, the entire fatigue characteristic must be applied, as follows

$$f(\tau_f,\mathbf{K}) = \tau_f(\mathbf{B},N_f) \quad \text{and} \quad f(\sigma_f,\mathbf{K}) = \tau_f(\mathbf{B},N_f). \tag{4}$$

Because the applied stress amplitudes τ_f and σ_f are functions of two fatigue characteristics $\tau_f(\mathbf{B}, N_f)$ and $\sigma_f(\mathbf{C}, N_f)$ the **K** set of constants takes the form of a set of functions

$$\mathbf{K}(\mathbf{B},\mathbf{C},N_f). \tag{5}$$

It makes the calculation of equivalent stress more complex, since the reducing function also becomes the function of N_f whose value is unknown. As result of this, it is necessary to solve the following equation with the purpose of calculating the adequate number of cycles to failure $N_{cal} = N_f$

$$f(\sigma_{ij}(t),n_i,s_i,\mathbf{K}(\mathbf{B},\mathbf{C},N_f)) = \tau_f(\mathbf{B},N_f). \tag{6}$$

Eq. (6) cannot be solved analytically; however it is a function of one variable N_f that could be found by implementing numerical methods. There could be a class of materials assigned to a given fatigue criterion for which **K** is constant. If **K** is only the function of stress ratio σ_f/τ_f (most cases) and the slopes of S–N curves σ_f and τ_f are equal then **K** is constant.

The above presented idea is not very new. This problem was already reported in [5,25–29]. However, its solution was not presented until the work in [30], in which a few multiaxial fatigue criteria were applied in order to calculate fatigue life of a few metallic materials under cyclic uniaxial and multiaxial proportional loading. A convergence of the revised algorithm and improvement of fatigue life calculation was demonstrated there. But still, several questions are open to answer. One of them is: is there a solution of Eq. (6) under non-proportional loading? Hence, one of the aims of this paper is to prove that such solution can be established whereas the other one is to show that improvement of fatigue life calculation is remarkable, compared to the classical algorithm. The scope of work reported here involves an analysis of two well-known multiaxial fatigue criteria and experimental data from three steels subjected to constant amplitude uniaxial bending, torsion and multiaxial proportional and non-proportional bending and torsion (all with zero mean values). The selected criteria are only exemplary. The revised algorithm could be implemented into different stress-based multiaxial fatigue criteria which apply material constants being the function of two fatigue limits: σ_{af} and τ_{af}. The problem of an adequate calculation of material parameters is important since the number of such types of fatigue criteria is large [11–14,22–24] and the new ones are still being proposed [31–34]. Further, the similar problem of relation between material constant and fatigue life is noticed in the Fatemi–Socie strain-based criterion [35] described in [29].

2. Experimental data

Experimental data were carefully selected in order to verify the efficiency of the revised algorithm. The three selected metallic materials differ in the values of the ratios of slope coefficients m_τ/m_σ of the S–N fatigue characteristics, i.e. fully reversed uniaxial cyclic torsion and bending. The gradual change in the m_τ/m_σ ratio allows verification of the improvement of fatigue life calculation by use the revised algorithm in comparison to the classical algorithm. All the experimental data include uniaxial torsion, bending and multiaxial proportional and non-proportional (δ phase shift equal to $\pi/2$) loading (torsion-bending). Such a collection of data provides grounds for determination of regression coefficients of the S–N curves by use of a single mathematical form and individual analysis of the revised and classical algorithms for each type of loading.

The applied S–N fatigue characteristics were determined according to the ASTM standard [36], i.e. under cyclic bending

$$\sigma_f(N_f) : \log N_f = A_\sigma - m_\sigma \log \sigma_f, \tag{7}$$

and torsion

$$\tau_f(N_f) : \log N_f = A_\tau - m_\tau \log \tau_f, \tag{8}$$

where N_f is the estimated number of cycles to failure, A_σ, m_σ, A_τ and m_τ are the parameters of a linear regression.

Table 1
Experimental data for the S355J2G3 steel subjected to non-proportional bending and torsion.

σ_a (MPa)	τ_a (MPa)	$N_{exp} \times 10^3$, cycles					
329	111	1359.6	584.4	348.0			
343	343	3349.2	327.6				
357	120	628.8	159.6				
380	128	103.2					
306	150	987.6	751.2	735.6	687.6	484.8	434.4
267	191	229.2	152.4				
238	170	1892.4	1760.4	1084.8	631.2	308.4	