International Journal of Fatigue 88 (2016) 178–184

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

A crack opening stress equation for in-phase and out-of-phase thermomechanical fatigue loading

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ARTICLE INFO

Article history:
Received 21 August 2015
Received in revised form 6 March 2016
Accepted 8 March 2016
Available online 15 March 2016

Keywords:
Fatigue crack closure
Thermomechanical fatigue
Strip yield model
Crack opening stress equation
Constraint

ABSTRACT

In this paper, a crack opening stress equation for in-phase and out-of-phase thermomechanical fatigue (TMF) loading is proposed. The equation is derived from systematic calculations of the crack opening stress with a temperature dependent strip yield model for both plane stress and plane strain, different load ratios and different ratios of the temperature dependent yield stress in compression and tension. Using a load ratio scaled by the ratio of the yield stress in compression and tension, the equation accounts for the effect of the temperature dependent yield stress and the constraint on the crack opening stress. Based on the scaling relation established in this paper, Newman's crack opening stress equation for isothermal loading is enabled to predict the crack opening stress under TMF loading.

1. Introduction

High temperature components made of nickel-based superalloys such as gas turbine blades or combustion chamber components are exposed to thermomechanical loadings, which can lead to crack initiation and crack growth. The phase angle between temperature and the mechanical loading varies depending on the location. From uniaxial strain controlled thermomechanical fatigue tests it is known that the phase angle has a strong influence on fatigue lives of polycrystalline nickel-based superalloys [1–4]. At high mechanical loadings, in-phase (IP, phase angle of 0°) TMF loading usually results in shorter fatigue lives than out-of-phase (OP, phase angle of 180°) TMF loading. This tendency is found to be reversed at low mechanical loadings. For phase-shift loading and especially a phase angle of ±90° fatigue lives are generally higher than for both IP and OP TMF-loading.

Due to the temperature dependence of the mechanical properties such as Young's modulus or the yield stress, strain controlled TMF tests show load ratios with either positive (OP loading) or negative mean stresses (IP loading). This is especially true for high mechanical strain amplitudes and large temperature ranges, where the stresses in tension and compression are bounded by the temperature dependent yield stress. In contrast to isothermal loadings, increasing mean stresses do not generally lead to shorter fatigue lives. Hence, it was concluded in [5] that the interaction between the temperature dependent mechanical properties and the history dependent mean stress evolution under thermomechanical fatigue loading has not been fully understood yet.

From a mechanism-based point of view, the influence of the mean stress on fatigue crack growth is explained with plasticity-induced fatigue crack closure. In fatigue crack growth models, typically an effective (reduced) stress intensity factor

$$\Delta K_{\mathrm{eff}} = \Delta K \cdot U. \tag{1}$$

is used to account for crack closure effects (see e.g. [6,7]), where ΔK is the range of the stress intensity factor and the function U describes the ratio

$$U = \frac{\sigma_{\infty,\max} - \sigma_{\mathrm{op}}}{\sigma_{\infty,\max} - \sigma_{\infty,\min}} = \frac{\Delta\sigma_{\mathrm{eff}}}{\Delta\sigma} = \frac{1 - \frac{\sigma_{\mathrm{op}}}{\sigma_{\infty,\max}}}{1 - R_\sigma}. \tag{2}$$

$\sigma_{\infty,\max}$ and $\sigma_{\infty,\min}$ denote the maximum and minimum stresses in a loading cycle, $\Delta\sigma$ is the stress range and σ_{op} is the crack opening stress. R_σ is the load ratio $\sigma_{\infty,\min}/\sigma_{\infty,\max}$. By correlating measured fatigue crack growth rates with ΔK_{eff} instead of ΔK, the influence of the load ratio on the fatigue crack growth rates can be compensated. Various empirical formulas have been proposed to calculate U as a function of the load ratio R_σ, see e.g. [7,8].

An alternative, analytical approach was proposed by Newman [9] on the basis of results obtained with a modified Dugdale model. The so called strip yield model (SYM) accounts for a growing

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http://dx.doi.org/10.1016/j.ijfatigue.2016.03.011
0142-1123/

fatigue crack, which leaves plastically deformed material behind the moving crack-tip. In the SYM, the crack opening stress is calculated from the contact stress profile at minimum load. In contrast to the empirical approaches, the SYM accounts for the complete loading history and is often used for the assessment of variable amplitude loading [10,11]. Based on the results of the SYM, Newman [12] developed a crack opening stress equation, which is capable of describing the influence of the load ratio R_σ, the ratio $\sigma_{\infty,\max}/\sigma_Y$ (σ_Y is the yield stress) and the constraint, i.e. plane stress or plane strain.

While crack opening stress equations as e.g. the Newman model are well established for isothermal loading conditions, an analytical crack closure model for nonisothermal conditions does not exist. Thus, the effect of crack closure cannot be reasonably taken into account in TMF applications, resulting in uncertain fatigue life predictions and misinterpretation of experimental results.

Experimental measurements of the crack closure effect under TMF loading can rarely be found in literature. In [13] the crack closure strain was measured using a potential drop method for Inconel 718. Analogously to Eq. (2) a function U was defined and evaluated for IP and OP TMF tests performed at different strain ratios. It was found, that U varies with the phase angle and the strain ratio. In [14] the effect of dwell times on the thermomechanical fatigue crack growth under IP TMF loading between $T = 50–550\,^\circ\mathrm{C}$ was studied for Inconel 718. The crack closure stress was evaluated for the single-edge notched specimen by monitoring the change of stiffness attributed to the point when the crack faces get into contact during unloading. With increasing dwell time the crack closure stress decreased significantly, but always stayed in the positive stress regime. The effect of IP and OP TMF loading on fatigue crack growth in IN792 was studied by [15] in the temperature range between $T = 100–750\,^\circ\mathrm{C}$ using a single-edge notched specimen. The IP TMF tests were conducted with a strain ratio of $R_\epsilon = 0$ while the OP TMF tests were performed with $R_\epsilon = -\infty$. The crack closure force detected from the change of stiffness similarly to [14] was always lower for OP TMF loading than for IP TMF loading.

In a recent paper [5] published by the authors, the SYM was modified to account for temperature dependent elastic and plastic material properties. For two different load ratios under plane stress conditions it was demonstrated, that plasticity-induced crack closure differs strongly for IP and OP TMF loading, as long as the temperature fluctuations go along with a significant change of the yield stress during a TMF cycle. With a mechanism-based lifetime model and the results of the modified SYM it was shown, that IP TMF tests have shorter lifetimes than OP TMF tests at high mechanical loadings and that the lifetime curves overlap at lower mechanical loadings. These effects can partly be explained by plasticity-induced fatigue crack closure.

It is the aim of this paper to develop a crack opening stress equation for TMF loading, which is valid for both plane stress and plane strain. To this end, the database for IP and OP crack opening stresses from [5] is extended using the temperature dependent SYM. For plane stress two additional load ratios are studied. For plane strain analogous calculations are performed for the same load ratios. The extension of the temperature dependent SYM to plane strain is presented in Section 2, while the results for all loading conditions are shown in Section 3. Based on the simulation results, a scaling relation is developed in Section 4. The scaling relation is able to describe all results obtained with the SYM independent of the phase angle, the constraint and the temperature dependent yield stress. The scaling relation is then used to modify the crack opening stress equation from Newman [12] to TMF loading. The temperature dependent SYM and the scaling relation with their limitations are discussed in Section 5 and concluded in Section 6.

2. Temperature dependent strip yield model for plane stress and plane strain

In the previous work [5] the analytical strip yield model from [9] for a center-cracked (infinite) plate with crack length $2a$ under mode I loading was extended to thermo-cyclic loading by accounting for the temperature dependent elastic and plastic material properties. However, solely plane stress conditions were investigated. Thus, some modifications for plane strain conditions are introduced next, before the plane strain SYM is validated. For the full details on the equations and implementation of the temperature dependent SYM, the reader is refered to [5].

For assessing plasticity-induced crack closure under plane strain conditions the constraint factors α and η according to [9] are set to $\alpha = 3$ and $\eta = \nu$, where ν denotes the Poisson's ratio. As described in [5], η enters into the calculation of the crack-surface displacements. The length of the plastic zone ω in front of the physical crack tip under thermomechanical fatigue loading and small scale yielding is computed by:

$$\omega = \left[a \left(\cos\left(\frac{\pi \sigma_\infty}{2\alpha\sigma_Y(T)} \right) \right)^{-1} - 1 \right], \tag{3}$$

where a is the physical crack length, σ_∞ is the outer applied stress and $\sigma_Y(T)$ is the temperature dependent yield stress. For $\alpha = 1$, i.e. plane stress conditions, the expression for ω given in [5] is obtained. In order to guarantee that Eq. (3) yields the maximum plastic zone size during a loading cycle, the maximum value of σ_∞/σ_Y within a loading cycle is used ($\sigma_\infty < \sigma_Y$).

The yield stress ratio $R_Y = -\sigma_{Y,c}/\sigma_{Y,t}$ defined in [5] ($\sigma_{Y,t}$ is the yield stress at the maximum applied load in tension, $\sigma_{Y,c}$ is the yield stress at the minimum applied load in compression), which will be used for the scaling relation derived in Section 4, now takes the form:

$$R_Y = \frac{\sigma_{Y,c}}{\alpha\sigma_{Y,t}}. \tag{4}$$

Thus, for isothermal loading, where $\sigma_Y = \sigma_{Y,c} = \sigma_{Y,t}$ is assumed, $R_Y = 1/\alpha$.

In order to validate the implementation of the SYM under plane strain (constraint factor $\alpha = 3$) conditions, the calculated crack opening stresses are compared in terms of U from Eq. (2) to the results of the crack opening stress equation of Newman [12]. To this end, temperature independent (i.e. constant) material properties are chosen.

The results are shown in Fig. 1 as a function of the normalized applied maximum stress $\sigma_{\infty,\max}/(\alpha\sigma_{Y,t})$. For all considered load ratios R_σ the results are in good agreement with the crack opening

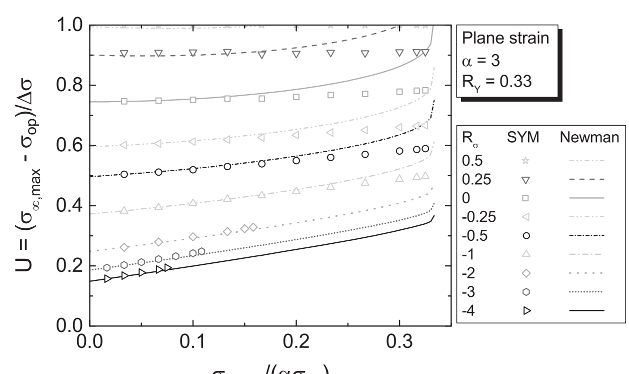

Fig. 1. Calculated crack opening stresses in comparison to the crack opening stress equation of Newman [12] for different load ratios and plane strain conditions.