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A strain energy based damage model for fatigue crack initiation and growth

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ABSTRACT

A strain energy based fatigue damage model is proposed which uses the strain energy from applied loads and the strain energy of dislocations to calculate stress-life, strain-life, and fatigue crack growth rates. Stress ratio effects intrinsic to the model are discussed, and parameterized in terms of the Walker equivalent stress and a fatigue crack growth driving force. The method is then validated using a variety of different metals with strain-life data and fatigue crack growth rate data available on the SAE Fatigue Design & Evaluation subcommittee database.

1. Introduction

The relationship between strain-life and fatigue crack growth has been formulated in different ways over the past few decades [1,2]. These methods employ the cyclic stress–strain relationship and strain energy for the purpose of bridging the connection. The strain energy density has also been used as a parameter for fatigue crack growth, or fatigue crack initiation, independently [3–5]. The strain energy of applied loads and the strain energy of dislocations have recently been related to fatigue crack initiation, in terms of the strain-life [6]. The work presented here intends to establish a generalized relationship between the strain energy density imparted by external loads, dislocation strain energy density, and fatigue life in the form of stress-life, strain-life, and fatigue crack growth. This work makes the following assumptions.

1. The cyclic Ramberg–Osgood stress–strain relationship, Eq. (1), is an appropriate description of the cyclic material behavior.
2. Fatigue damage can be related to an increase in tensile strain, and compressive strains do not independently contribute to fatigue damage.
3. The peak tensile stress, σ_{max}, must be greater than 0 in tension in order for fatigue damage to occur.
4. There is some critical dislocation density, ρ_c, which does not depend on the cyclic strain range or peak stress.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \tag{1}$$

In the Ramberg–Osgood equation, Eq. (1), ε is the strain, σ is stress, E is the elastic modulus, K' is the cyclic strength coefficient, and n' is the cyclic strain hardening exponent.

The proposed general fatigue damage model is

$$\left(\frac{U_e}{U_d\rho_c}\right)\left(\frac{U_p^*}{U_d\rho_c}\right) = D = \frac{2N}{2N_f} \tag{2}$$

where U_e is the tensile elastic strain energy density from σ_{max}, U_p^* is the complimentary plastic strain energy density from $\Delta\sigma, U_d$ is the dislocation strain energy, ρ_c is the critical dislocation density, D is the fatigue damage, $2N$ is the number of reversals per tensile reversal which lead to the strain energy densities U_e and U_p^*, and $2N_f$ is the number of reversals until a fatigue crack has initiated. U_d can be approximated as

$$U_d \approx G\vec{b}^2 \tag{3}$$

where $\vec{b}$ is the burgers vector, the size of the diameter of an atom of the primary constituent for most metals and alloys [7], and

$$G = \frac{E}{2(1+\nu)} \tag{4}$$

where G is the shear modulus and ν is Poisson's ratio.

Prior work in strain energy density based fatigue modeling has either summed the elastic and plastic components of strain energy

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density, or ignored the elastic component due to its magnitude relative to plastic strain energy density at higher strains. This work differs in that it uses the *complimentary* plastic strain energy density, and includes both elastic and plastic components, but does not sum the elastic and plastic strain energy density terms.

2. Stress-life/strain-life

The strain energy densities U_e and U_p^* can be calculated by integrating the cyclic Ramberg–Osgood equation, in the form

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (5)$$

which gives

$$\int_0^{\Delta\sigma} \Delta\varepsilon d\Delta\sigma = \frac{\Delta\sigma^2}{2E} + 2\left(\frac{n'}{1+n'}\right)\left(\frac{1}{2K'}\right)^{(1/n')} (\Delta\sigma)^{(1+n')/n'}. \quad (6)$$

The tensile elastic strain energy density is related to the peak tensile stress, not the change in tensile stress, and so σ_{max} is substituted for $\Delta\sigma$ in the case of the elastic strain energy density, which gives

$$U_e = \frac{\sigma_{max}^2}{2E}. \quad (7)$$

The complimentary plastic strain energy density is

$$U_p^* = 4\left(\frac{n'}{1+n'}\right)\left(\frac{1}{K'}\right)^{(1/n')} (\sigma_a)^{\frac{(1+n')}{n'}} \quad (8)$$

where σ_a is the stress amplitude, which is equivalent to $\Delta\sigma/2$. Substituting Eqs. (7) and (8) into Eq. (2), and accounting for 2 reversals for every damage causing reversal, yields

$$\left(\frac{2}{U_d^2\rho_c^2 E}\right)\left(\frac{n'}{1+n'}\right)\left(\frac{1}{K'}\right)^{(1/n')} \times (\sigma_{max}^2)(\sigma_a)^{\frac{(1+n')}{n'}}\left(\frac{1}{N}\right) = \frac{1}{2N_f}, \quad (9)$$

which gives the stress life. This can be related to the elastic strain at fully reversed loading by making the substitution

$$\sigma = \varepsilon_e E, \quad (10)$$

where ε_e is the elastic strain, which leads to a power law relationship that can be related to the elastic part of the Basquin–Manson–Coffin [8–10] equation,

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \quad (11)$$

where σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient, and c is the fatigue ductility exponent, where

$$\sigma'_f = E \times \left(\frac{2}{U_d^2\rho_c^2}\left(\frac{1}{K'}\right)^{(1/n')}\left(\frac{n'}{1+n'}\right)E^{(1+2n')/n'}\right)^{\frac{-n'}{1+3n'}} \quad (12)$$

and

$$b = \frac{-n'}{1+3n'}. \quad (13)$$

Similarly, substituting for plastic strain using

$$\varepsilon_p = \left(\frac{\sigma}{K'}\right)^{1/n'} \quad (14)$$

where ε_p is the plastic strain, yields

$$\varepsilon'_f = \left(\frac{2(K')^3}{U_d^2\rho_c^2 E}\left(\frac{n'}{1+n'}\right)\right)^{\frac{-1}{1+3n'}} \quad (15)$$

and

$$c = \frac{-1}{1+3n'}. \quad (16)$$

It is well known that combining Eqs. (5) and (11) the relationships

$$\frac{b}{c} = n' \quad (17)$$

and

$$K' = \frac{\sigma'_f}{\left(\varepsilon'_f\right)^{n'}} \quad (18)$$

should be true for a well behaved material. Using Eqs. (13) and (16), it is found that

$$\frac{b}{c} = \frac{\frac{-n'}{1+3n'}}{\frac{-1}{1+3n'}} = n' \quad (19)$$

and using Eqs. (12) and (15) it is found that

$$K' = \frac{\sigma'_f}{\left(\varepsilon'_f\right)^{n'}} = \frac{E \times \left(\frac{2}{U_d^2\rho_c^2}\left(\frac{1}{K'}\right)^{(1/n')}\left(\frac{n'}{1+n'}\right)E^{(1+2n')/n'}\right)^{\frac{-n'}{1+3n'}}}{\left(\left(\frac{2(K')^3}{U_d^2\rho_c^2 E}\left(\frac{n'}{1+n'}\right)\right)^{\frac{-1}{1+3n'}}\right)^{n'}} \quad (20)$$

both of which are true equalities in this model. The value of ρ_c can be found by fitting the resultant strain-life curve to low cycle strain-life data by solving Eq. (9) for ρ_c,

$$\rho_c = \sqrt{\left(\frac{2(2N_f)}{U_d^2 EN}\right)\left(\frac{n'}{1+n'}\right)\left(\frac{1}{K'}\right)^{(1/n')} \times (\sigma_{max}^2)(\sigma_a)^{\frac{(1+n')}{n'}}}, \quad (21)$$

which can be used as is, or the stress can be substituted with elastic or plastic strain by using the elastic or plastic term in the Ramberg–Osgood equation. It has been found that, generally, $1 \times 10^{15} \leqslant \rho_c \leqslant 3 \times 10^{16}$ m^{-2}. Note that in this model a perfectly plastic material, where $n' = 0$, does not accumulate fatigue damage. A comparison of the results of this model with strain life measurements are shown in Figs. 1–17, and the values used and calculated are listed in Table 1.

The effect of the stress ratio on the strain life in this model is shown in Fig. 18. Examination of Eq. (9) yields the same stress ratio effect proposed by Walker, in Ref. [24], shown in Eq. (22).

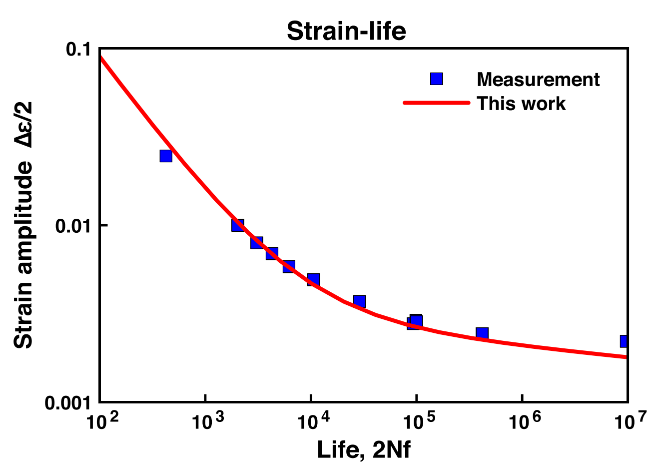

Fig. 1. 10B20 Boron steel adapted from the SAE FD&E database, originally from Ref. [11].