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Staggered Padé wavelength distribution for multi-Bragg photonic mirrors

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ABSTRACT

High reflective mirrors are commonly used in solar concentration devices. It has been shown that photonic mirrors are suitable for this purpose. However, despite their high reflectivity, they absorb radiation and heat up showing degradation. To overcome this problem here we propose to enhance the reflectivity of the mirrors. We report an improved and easy method to design broadband high reflective mirrors and build them from porous silicon multilayers. Those are composed of a continuous arrangement of submirrors reflecting each one at a given wavelength. This method consists in staggering the wavelength distribution following a relation based on the Padé approximant. We simulate the reflectance spectra using the transfer matrix method taking into account the experimental complex refractive index of porous silicon. The comparison between the experimental and the reflectance spectra shows a good agreement. With this technique we have fabricated high quality dielectric mirrors that can be used for concentrated solar energy applications among others.

1. Introduction

Sunlight as a clean energy source can contribute considerably to the solution of the energy problem if appropriate methods are developed to collect, concentrate, store and convert solar radiation. In the particular case of solar concentration devices, materials with high reflectivity are needed. The materials commonly used as solar reflectors are silver-coated glass and aluminized reflectors [1]. However recently photonic crystals have been proposed [2] due to its diverse optical properties such as high reflectivity and tuning of the band gap. Unlike metallic reflectors, photonic mirrors are selective and reflect at a specific wavelength without the interference of unwanted wavelengths. De la Mora et al. reported that despite their high reflectivity they absorb radiation and present a significant thermal degradation when exposed to concentrated solar radiation. To surpass this problem, here we propose the enhancement of the reflectivity of the mirrors.

It has been shown that porous silicon (PS) is a suitable material for the fabrication of photonic mirrors [3]. PS has many optical applications [4], such as filters [3], sensors [5], waveguides [6] and photonic oscillators [7]. It is an excellent material to produce one-dimensional photonic crystals, like dielectric Bragg reflectors [8,9]. These are structures formed from multiple layers of different refractive index materials that reflect the light around a central wavelength. When PS multilayers have a high index contrast between layers there is an increase in the photonic quality [10].

As our goal is to make PS mirrors to reflect the solar spectrum, we need to consider the absorption coefficient of PS in the visible region [11]. The complex refractive index of PS was calculated by spectroscopic ellipsometry and included in our reflectance spectra calculations. Several authors [9,12,13] have reported theoretical studies of enlarged reflection range in PS multilayered structures, but they do not present experimental validation. Estevez et al. [14] have shown a refined technique to fabricate PS mirrors, simulated however without considering a complex refractive index.

PS broadband mirrors can be constructed as a continuous arrangement of Bragg submirrors where each one reflects around a central wavelength and the superposition of submirrors covers a large range of frequencies due to the enlargement of the photonic band gap. The range of wavelengths to be reflected depends on the wanted application, in the case of mirrors for solar concentration it covers from visible to IR. The central wavelength distribution ($\lambda(s)$) can be selected by a given function that depends on the submirror number (s) and the number of layers. Several authors have proposed different ways to calculate the submirror wavelength distribution by a recursive formula [2], following a geometric sequence [15], or by chipred grating [16]. However, it is always

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necessary to establish a selection method for the central wavelengths to reach the maximum reflectance. It is important to stress that all these methods can be used to design dielectric mirrors in general, not only using PS.

In this work we implemented a new optimized procedure to calculate the central wavelength distribution $\lambda(s)$ of the submirrors in order to construct high reflectance broadband mirrors. This method is based on a Padé approximant, a mathematical technique widely used to approach rational functions constructed from the coefficients of the Taylor series expansion [17]. The Padé approximant often gives the best approximation to the function because of its convergence properties [18]. It provides an approximation throughout the whole complex plane and therefore has a wide applicability in different areas of knowledge [19]. Here the so-called Padé wavelength distribution is constructed once the required reflectance range is fixed and a discrete number of solutions to $\lambda(s)$ are determined. We simulated the reflectance spectra corresponding to each distribution and chose the one with the highest reflectivity. We fabricated high reflective PS mirrors designed with this method and compared their reflectance spectra with the simulated model, which considers absorption. The comparison shows a general good agreement between experiments and theory but there are small differences that need to be analyzed. We explore some physical explanations and validate them using a merit function.

In the next section we present the transfer matrix method that is used to simulate the reflectance spectra considering absorption. In Section 3 we describe the construction of the wavelength distribution using the Padé approximation. With this input we are able to describe the fabrication procedure in Section 4. Subsequently we present experimental results in Section 5 and perform a comparison with our theoretical model including absorption. Our final remarks are shown in Section 6.

2. Theoretical reflectance spectrum considering absorption

The reflectance spectrum of a multilayer can be calculated using the transfer matrix method [20]. According to this theory electromagnetic field propagation in a multilayer is represented by a matrix

$$\begin{bmatrix} E_l \\ H_l \end{bmatrix} = \mathbf{M} \begin{bmatrix} E_{(l+1)} \\ H_{(l+1)} \end{bmatrix},$$

where E_l, H_l, $E_{(l+1)}$, $H_{(l+1)}$ are the electrical and magnetic fields in the first and the last interface, respectively. Here l represents the number of layers in the structure. The transfer matrix $\mathbf{M}$ is given by the product

$$\mathbf{M} = \mathbf{M}_{d_1}\mathbf{M}_{d_2}\ldots\mathbf{M}_{d_l},$$

where $\mathbf{M}_{d_j}$ are the characteristic matrices of each layer from $j = 1, 2, \ldots l$. For a transverse electric field (TE) at normal incidence

$$\mathbf{M}_{d_j} \equiv \mathbf{M}_{d_j}(n_j, \lambda) = \begin{pmatrix} \cos\left(\frac{2\pi n_j d_j}{\lambda}\right) & i\sin\left(\frac{2\pi n_j d_j}{\lambda}\right)/Y_j \\ Y_j i\sin\left(\frac{2\pi n_j d_j}{\lambda}\right) & \cos\left(\frac{2\pi n_j d_j}{\lambda}\right) \end{pmatrix}.$$

The refractive index and the thickness in layer j are n_j and d_j respectively. Y_j is determined by $Y_j = \sqrt{(\epsilon_0/\mu_0)}n_j$, where ϵ_0 and μ_0 are the vacuum permittivity and the permeability respectively λ is the wavelength.

The reflection coefficient can be written in terms of the elements m_{ij} of $\mathbf{M}$, here Y_a stands for the incident media (air)

$$r = \frac{Y_a m_{11} + Y_a Y_{(l+1)} m_{12} - m_{21} - Y_{(l+1)} m_{22}}{Y_a m_{11} + Y_a Y_{(l+1)} m_{12} + m_{21} + Y_{(l+1)} m_{22}},$$

from here the reflectance is obtained as

$$R = |r|^2.$$

We are interested in a multilayer constructed by f number of submirrors, with p being the number of periods composed of layers of high (H) and low(L) refractive indices. Hence, the total transfer matrix for this structure is given by

$$\mathbf{M} = (\mathbf{M}_{d_H(1)}\mathbf{M}_{d_L(1)})^p(\mathbf{M}_{d_H(2)}\mathbf{M}_{d_L(2)})^p\ldots(\mathbf{M}_{d_H(f)}\mathbf{M}_{d_L(f)})^p. \quad (1)$$

The total transfer matrix is constructed by multiplying iteratively the submirror matrices, Eq. (1), and then the reflectance can be computed. The reflectivity in multilayered structures is usually simulated considering only the real part of the complex refractive index $\eta(\lambda)$, therefore the absorption is not taken into account. However in the wavelength region from 300 to 1000 nm, porous silicon layers show high absorption [11]. Thus we performed complete theoretical spectra of our PS mirrors including this property. This is estimated by introducing a complex refractive index in the elements of the transfer-matrix ($\eta(\lambda) = n(\lambda) - ik(\lambda)$), where k is the extinction coefficient. In this way the reflectance is computed as a function of $\eta(\lambda)$ and λ.

One reason that has obstaculized considering the absorption is that the transfer matrix becomes numerically unstable for short wavelengths or large thicknesses when the complex refractive index is included in the calculations [21–23]. The set of instabilities that appear in the simulations is called the ωd-problem and emerges when the transfer matrix elements become real exponentials. Among others, a cause for this instabilities is to consider a complex refractive index. Besides absorption, the numerical instability also exhibits when evanescent waves are present in the dielectric layers [24]. As the absorption coefficient of porous silicon increases sharply for wavelengths below 500 nm, we evaluate the possibility of having numerical instabilities in that region. To determine if this problem affected our calculations of the reflectance spectra we compared the simulations obtained from the transfer matrix method with a hybrid matrix $\mathbf{H}$ calculation. This alternative method is numerically stable in any region and thickness [23,25]. The spectra obtained from both methods show no significant differences between each other, since in this case the reflectance is not affected, therefore we are confident of our results.

In the next section we describe the method we implemented based on the Padé approximant to determine the submirror wavelength distribution for a multilayered structure.

3. Padé wavelength distribution approximation

Padé approximants are typically used when there is some unknown underlying function. In this technique not only the approximant's power series agrees with the power series of the function, it can also provide new data about the function. This method has proved to be very useful in providing information about the solution of many interesting problems in applied sciences. It is viewed as a method to an approximate analytic continuation [26], the range of convergence of the Padé approximants is not as restricted as the Taylor series and it may often give a better approximation to the function. A Padé approximant is defined as the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function. Considering a given power series