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# Fatigue life assessment of steel samples under various irregular multiaxial loading spectra by means of two energy-based critical plane damage models

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A B S T R A C T

Fatigue life of different steel alloys undergoing multiaxial irregular loading spectra was evaluated based on two energy-based models of Łagoda–Macha (LM) and Varvani-Farahani. The LM damage model evaluated the life of samples from calculation of the equivalent strain energy densities over counted reversals of the applied stress and strain histories on the critical plane. The Varvani damage approach assessed fatigue life through integration of the normal and shear energy ranges calculated on the critical plane at which the largest stress and strain Mohr's circles over the counted loading and unloading reversals were determined. Based on the equivalent relative strain method of the Wang–Brown, peaks and valleys (reversals) were counted over irregular multiaxial loading spectra. Damage values were calculated and then accumulated over peak–valley events of a block loading spectrum. The overall damage over block histories was then related to fatigue life $N_f$ in the right hand side of the damage models. The predicted lives based on these damage models were compared with those of reported experimental data. The choice of damage assessment models and how to determine the fatigue life of components under irregular loading spectra were discussed.

## 1. Introduction

Design of engineering components and structures such as pressure vessels, turbines, airframes and landing gears undergoing irregular loading requires special attentions to increase service safety and prevent mechanical failure as well as decrease the cost of maintenance. It is essential to develop capable models to assess fatigue life under complex states of stress and strain [1]. For the evaluation of fatigue life of components under multiaxial loading conditions, different approaches have been implemented. Critical plane approach was developed based on the empirical observations of crack nucleation and early growth under applied stress cycles. In addition to evaluate the fatigue life of components, the models based on the critical plane approach are able to identify the orientation of dominant failure planes. In this approach, crack propagation in specific plane which is called critical plane is regarded as a main contributor to the fatigue damage process taking place under different conditions. Stress-based models have been successfully employed in the high-cycle regime when the plastic strain is small [2–7]. In the low-cycle fatigue regime, life data are associated with plastic strain amplitude and a power law relation between plastic strain and fatigue life was introduced by Coffin–Manson equation [8,9] in early sixties. Strain-based parameters became widely popular in finite life design strategy of component [8–13]. Fatemi and Socie [11] involved normal stress along with shear strain components acting on the critical plane to further extend the capability of the model for both low- and high-cycle fatigue regions. Energy-based approach was proposed based on the irreversible process of plastic deformation in each cycle with association of dissipation of strain energy in microscopic level. The dissipated energy over cycles was then related to the fatigue damage over life cycles. Unlike stress- and strain-based parameters, energy-based criteria addressed interaction between stress and strain in deformation process and reflected path dependency of material response [14–16]. Researchers [12,17–25] further developed damage criterion on the basis of strain energy density on the critical plane. In this approach, fatigue damage assessment involved components of stress and strain on the critical plane to address the shortcomings of earlier developed strain-based and stress-based models. Smith–Watson–Topper (SWT) [17] developed simple equation multiplying principal strain range and maximum stress on the principal strain range plane. Liu [19] proposed virtual strain energy model based on the elastic and

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**Nomenclature**

| | |
|---|---|
| $b$ | axial fatigue strength exponent |
| $c$ | axial fatigue ductility exponent |
| $b_0$ | shear fatigue strength exponent |
| $c_0$ | shear fatigue ductility exponent |
| $E$ | elastic modulus |
| $G$ | elastic shear modulus |
| $\hat{l}_n,\ \hat{m}_n,\ \hat{n}_n$ | mean direction cosines of principal stress/strain |
| $N_f$ | fatigue life |
| $\bar{n}$ | unit normal vector |
| $Q$ | fatigue limit |
| $\bar{s}$ | mean direction of maximum principal stress |
| $W_n$ | normal strain energy density |
| $W_{ns}$ | shear strain energy density |
| $W_{eq}$ | equivalent strain energy density |
| $\beta$ | material constant |
| $\kappa$ | material constant |
| $v$ | Poisson's ratio |
| $v'$ | effective Poisson's ratio |

| | |
|---|---|
| $\varepsilon'_f$ | axial fatigue ductility coefficient |
| $\varepsilon_n$ | normal strain on the critical plane |
| $\varepsilon_{ij}$ | strain tensor (where $i$ and $j$ = 1, 2, 3) |
| $\varepsilon^*_{eq}$ | equivalent relative strain |
| $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$ | principal stresses ($\varepsilon_1 < \varepsilon_2 < \varepsilon_3$) |
| $\gamma_{ns}$ | maximum shear strain on the critical plane |
| $\gamma'_f$ | shear fatigue ductility coefficient |
| $\Delta(\gamma/2)_{\max}$ | shear strain range acting on the critical plane |
| $\Delta\sigma_n, \Delta\varepsilon_n$ | normal stress and strain ranges acting on the critical plane |
| $\sigma_1,\ \sigma_2,\ \sigma_3$ | principal stresses ($\sigma_1 < \sigma_2 < \sigma_3$) |
| $\sigma_n$ | normal stress on the critical plane |
| $\sigma$ | normal stress |
| $\sigma'_f$ | axial fatigue strength coefficient |
| $\Delta\tau_{\max}$ | shear stress range acting on the critical plane |
| $\tau_{ns}$ | maximum shear stress on the critical plane |
| $\tau'_f$ | Shear fatigue strength coefficient |
| $\tau$ | Shear stress |

plastic work components. Chu et al. [12] modified this model to include the influence of mean stress using maximum stress instead of stress range. Glinka et al. [20] employed maximum normal and shear stress to account for the mean stress effect on crack growth. The normal stress on the critical plane was responsible to open the crack and the maximum shear stress to overcome any sliding friction occurring between the crack surfaces. Łagoda and Macha [25] developed a criterion based on the maximum shear and normal stress in the fracture plane. For multiaxial stress cycles, the equivalent stress history is employed to assess fatigue damage over high-cycle fatigue region [26] while for the strain-based multiaxial fatigue cycles, fatigue life was related to a linear combination of maximum shear strain and normal strain acting on the fracture plane respectively coinciding with mean direction of maximum shear strain and its normal direction [25,27]. Łagoda and Macha further developed generalized criterion on the maximum shear and normal strain energy density parameter on the critical plane [25]. They verified proposed stress-based criteria to estimate fatigue life of 30CrNiMo8 steel alloy under multiaxial random loading. Lives were predicted within the scatter band of factor 3 [26]. They also calculated fatigue life data of steel alloys under uniaxial and multiaxial random loading conditions using energy-based model [25,28–31]. Varvani-Farahani [24] proposed a damage model integrating normal and shear strain energy ranges determined from stress and strain components acting on the critical plane. The model was employed to assess life of various materials under uniaxial and multiaxial loading spectra by several researchers. Han et al. [32] employed the Varvani model to assess fatigue life of SNCM630 steel alloy under proportional and non-proportional loading conditions. Their predicted life data fell within factor 2. Chen et al. [33] utilized the Varvani damage model to evaluate fatigue life of Al 7050-T7451 and En15R steel alloys under combined tension–torsion variable loading histories and reported the predicted values fall within the scatter band of factor 2 as compared with experimental data. In a recent paper [34] fatigue damage and life of steel samples subjected to random loading conditions were evaluated based on the SWT, Ellyin, Łagoda–Macha and Varvani damage models with different energy-based descriptions. Fatigue lives based on Varvani and Macha tightly fell in agreement with those of experimental data as compared with two other damage models.

The present study intends to assess fatigue life of steel alloys undergoing multiaxial irregular loading paths based on critical plane-energy based damage descriptions developed earlier by Varvani and Łagoda and Macha (LM). Multiaxial cycle-counting of Wang–Brown was employed to determine components of normal stress and shear stress and the corresponding maximum dissipated energy acting on the critical plane over peak–valley events of the loading spectrum. They proposed equivalent energy parameter to include the normal and shear strain energy densities in the critical plane and the direction of this plane is evaluated based on the weight function method. While, the Varvani's model related fatigue damage to life of steel alloys through components of normal and shear stress/strain acting on the critical plane. The choice of damage approaches are discussed based on the stress/strain components they hold and their degree of complexity and consistency in damage assessment.

## 2. Energy based damage models

### 2.1. Łagoda-Macha damage model

Macha developed a stress-based criterion [35] involving equivalent stress calculated from the maximum shear and normal stress acting on the critical plane to evaluate fatigue damage under multiaxial random loading conditions. The model [25,29–31] was further developed to evaluate fatigue life in low- and high-cycle fatigue regimes through the components of maximum normal and shear energy densities defined on the critical plane:

$$\max\{\beta W_{ns}(t) + \kappa W_n(t)\} = Q \qquad (1)$$

In this model, the critical plane is defined perpendicular to the mean direction along with maximum principal stress. The plane was identified by the unit normal vector $\bar{n}$ coinciding with the mean direction of stress $\bar{s}$. The mean direction coincided with the maximum shear strain energy density $W_{ns}(t)$ and the normal energy density $W_n(t)$ occurred along unit normal vector $\bar{n}$. Coefficients $\beta$ and $\kappa$ in Eq. (1) are material constants and are defined respectively through $\beta = 2/(1+v)$ and $\kappa = 2/(1-v)$. Left hand side of Eq. (1) is presented in short form of max $\{W_{eq}(t)\}$ and fatigue fracture takes place when maximum value of $W_{eq}(t)$ exceeds uniaxial fatigue limit $Q$. To calculate equivalent energy, the critical