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A log-normal format for failure probability under LCF: Concept, validation and definition of design curve

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ABSTRACT

Turbine components are usually designed onto safe-life approach, where the low-cycle fatigue analysis is based on design life curves with suitable probabilistic life margins. However, in order to design for a given reliability, the definition of the design curve should not only include the life variability but also the scatter of applied load. Unfortunately, in the literature there are few indications which only refer to safety factors under HCF, without any specific discussion for the case of components subjected to LCF.

In this paper, we firstly propose a log-normal format for calculating reliability for an assessment point $(\hat{\epsilon}, \widehat{N})$ based on a first order approximation. The validity of the approach is then proved for two different materials with a series of Monte Carlo simulations, where the material cyclic response is coupled to its $\epsilon - N$ diagram. The format is then used for estimating failure probability and for defining the design point which corresponds to a target failure probability. A safety factor is then proposed and its application to a series of steels for power generation is shown.

1. Introduction

Turbine components (rotor disks, blades, blade attachments) are heavy duty components classified as safety critical components. From the point of view of engineering design, the *safe-life* method [1] is the first step for defining morphology and material of the components, whose life issues and structural integrity are often re-analyzed with *damage tolerance* and probabilistic approaches into the subsequent assessment phase. Considering components that are critical for the safety, it is important to adopt a probabilistic approach for design and assessment. In the case of structures subjected to static loads (see [2]), the design is based onto probabilistic concepts for material properties (static strength) and loads (permanent loads, variable loads) in order to obtain target failure probabilities of the order of 10^{-5}. In particular, the standards adopt a semi-probabilistic approach and they provide the designer a series of *partial safety factors*, which have to be applied to the *characteristic values* of loads and resistance in order to achieve the target reliability.

If these indications are very detailed for static analyses (e.g. see BS7910 for static assessment with flaws [3]), on the other hand there are very few indications about the choice of safety factors in *safe-life* and they only refer to high cycle fatigue. In particular, Eurocode 3 [4] prescribes the adoption of S–N diagrams for different welded details which correspond to a 5% failure probability: the designer for the assessment has to apply a *safety factor* $\gamma_{Mf} = 1.35$ (for safety critical components) onto the S–N curve of the welded detail (see Fig. 1a, where the *knee* of the curve is N_D, S_D). A similar proposal, but non limited to welded details, is the one of the *FKM Guideline* [5], which prescribes S–N diagrams with $P_f = 2.5\%$ (i.e. $\mu - 2\sigma$) and a *safety factor* equal to 1.5 for steel and aluminium components (for safety critical components). However, these prescriptions do not precisely mention how the load/stress dispersion has to be considered and they do not allow to design for a given *target* failure probability.

As for nuclear components, ASME Boiler & Pressure Vessel Design Code Sect. III-Div. 1 [6] prescribes design curves obtained from the best fit of experimental data (strain-controlled tests on small specimens) [7] by applying a factor of 2 on strain and a factor of 20 on life, whichever the more conservative. These generous factors are not *safety margins* but rather adjustments for significant effects (scatter, surface finish, size) that can be expected in assessment of reactor components [7,8].

In the case of aeroengine components, according to [9], the traditional approach is based onto *life curve* corresponding to a 10^{-3}

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Nomenclature

b	fatigue strength exponent
c	fatigue ductility exponent
e	local strain
s	local stress
f_X	probability density function for the X variable
CV_X	coefficient of variation of the X variable
E	elastic modulus
F_X	cumulative distribution function for the X variable
K', n'	parameters of Ramberg–Osgood equation for cyclic response
K_t	stress concentration factor
HCF	high cycle fatigue
HT	high temperature
L	load
LCF	low cycle fatigue
N	number of cycle
$\widehat{N}$	target design life
N_{life}	prospective life considering the variability of $\hat{\epsilon}$
P_f	probability of failure
$r.v.$	random variable
R	resistance
RT	room temperature
S	nominal stress
$\widehat{S}$	nominal stress amplitude at design point
$\overline{X}$	sample mean for the X variable
$\mathbf{X}$	random variable X (**bold character**)
β	safety margin
ϵ	strain
ϵ'_f	fatigue ductility coefficient
$\hat{\epsilon}$	design local strain
ϵ_R	strain resistance at the number of cycles $\widehat{N}$
η	efficiency of first order estimates for dispersion (for strain and prospective life)
γ_F	partial safety factor to be adopted for $\epsilon - N$ diagram
γ_{Mf}	partial safety factor for S–N diagram adopted by Eurocode
μ_X	mean value of r.v. X
σ'_f	fatigue strength coefficient
$\sigma_{\log N}$	standard deviation of log-lives along the $\epsilon - N$ diagram
σ_X	standard deviation of r.v. X
ΔS	nominal stress range
Φ	cumulative Gaussian standard distribution function

failure probability, namely the curve $\mu - 3\sigma$ obtained from experiments is taken as the reference for life assessment [10]. It means that the *safe life* method, since it does not consider any variability in applied stress (or applied local strain ϵ), needs some other safety factor for achieving the target reliability.

1.1. Probabilistic analysis under LCF

As for the probabilistic analysis under LCF, Ellingwood [11] proposed the adoption of a first order approximation for estimating life dispersion of welds but he did not clarify how the variability of material response has to be considered. Wirsching [12] discussed how to handle life distribution in terms of a lognormal distribution dependent on plastic strains without any correlation to stress variability that, instead, was thoroughly discussed for the HCF regime. In [13] the authors adopted a 10% scatter in strain amplitudes for estimating life distribution, without any specific discussion about their choice for the strain dispersion. Socie [14,15] provided a complete description of the probabilistic model for LCF and how to estimate the prospective life distribution for a notched member with a Monte Carlo simulation. More recent papers develop a probabilistic model based on Weibull distribution for modelling the dependence of LCF life on size [16,17].

However, all the previous approaches are based onto the estimation of the distribution of the prospective life N_{life} (see Fig. 2a), so that the failure probability is:

$$P_f = \Pr[\mathbf{N}_{\text{life}} < \widehat{N}] \tag{1}$$

From a safety critical design prospective, we are more interested in the probability of failure at the design life. For example, what is the probability of failure in, say, 3000 startup-shutdown cycles of a turbine. This condition is shown in Fig. 2b. Each part in the population will have a fatigue strength or resistance described by the distribution $\boldsymbol{\epsilon_R}$. Similarly, each part in the population will have a loading distribution (in terms of strain) described by $\hat{\epsilon}$. In this case the failure probably is given by:

$$P_f = \Pr[\hat{\boldsymbol{\epsilon}} > \boldsymbol{\epsilon}_R] \tag{2}$$

at a particular lifetime $\widehat{N}$.

1.2. Paper overview

In the first part of the paper we firstly propose a simple format based on *first order* approximation for calculating reliability (or failure probability) for a given assessment point $(\hat{\epsilon}, \widehat{N})$ and we provide a closed form solution to the procedure adopted in [14]. In particular, we will propose a new *log-normal format* for calculating failure probability in terms of *interference* between the distributions $\hat{\boldsymbol{\epsilon}}$ and $\boldsymbol{\epsilon_R}$, the strain at which the material can resist for $\hat{N}$ cycles. This will allow us to discuss the definition of a simple safety factor for a *safe-life* design with a target reliability.

The plan of the paper is as follows. Section 2 shows the need of adopting adequate safety factors for a safe design. In Section 3 we propose a simple first order approximation for calculating the prospective distribution of fatigue life corresponding to a given design strain $\hat{\epsilon}$ and the calculation of failure probability. Then, in Section 4 the simple method is successfully compared with results obtained with a series of Monte Carlo simulations based on two different steels, whose properties were obtained at RT and HT. In Section 5 we then discuss the derivation of design curves, we propose the adoption of a γ_F safety factor to be applied to $\epsilon - N$ diagram and its application to some typical rotor steels.

2. Calculation of failure probability with Gaussian distributions

Let us consider two Gaussian distributions, one for the *load* **L** and another one for the *resistance* **R**. The failure probability, namely $\Pr\{L > R\}$ can be calculated as [18]:

$$P_f = \Pr[\mathbf{L} > \mathbf{R}] = \int_0^\infty f_L(l) \cdot F_R(l) dl \tag{3}$$

In the case the two distributions are Gaussian, then the calculation can be simplified with:

$$P_f = \Pr[(\mathbf{R} - \mathbf{L}) < 0] = \Phi(-\beta) \tag{4}$$

where the *safety margin* is: