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Application of the cyclic R-curve method to notch fatigue analysis

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ABSTRACT

The paper deals with the prediction of the fatigue limit of notched specimens. Some authors in the literature have shown that this issue can be solved by employing an approach based on critical distances, others used elastic fracture mechanics. In this paper the fatigue limit of notched specimens is given by the non-propagating condition of mechanically small surface cracks. According to the so-called cyclic R-curve method, the crack driving force of a growing small crack is compared to its resistance force, which incorporates the gradual build-up of crack closure. The fatigue limit is determined by that applied nominal stress, for which the tangency condition of crack driving and resistance force is satisfied. The approach has been modified to incorporate plasticity effects in the mechanically short crack regime and it has been applied to a mild and a high-strength steel, in case of an infinite plate with circular hole under remote tensile stress. The method has been successfully applied to the evaluation of fatigue notch-sensitivity in case of surface roughness.

1. Introduction

Notch fatigue analysis is of primary importance in the fatigue design of many engineering components, as fatigue damage takes place often at stress raisers (for instance shoulder fillets in shaft and axles, threaded connections, welded joints, etc.).

Many authors dealt with this issue and proposed different approaches to the aim of predicting the fatigue limit of a given material in presence of notches. The methodologies can be classified under two main headings: (i) critical distance or stress based approach and (ii) fracture mechanics based approach.

In general, the critical distance method is based on the concept that it is not the maximum stress at the notch root the important parameter controlling the fatigue limit, but a certain stress value beyond the notch root. Already the pioneering works by Neuber [1] and Peterson [2] hypothesized that the controlling damage parameter is an average stress over a characteristic structural length or a punctual stress at a given distance from the notch root, respectively.

A major work about critical distances has been presented by Taylor in his unifying theoretical model [3,4]. The underlying assumption of his work has been that the average elastic stress must exceed the fatigue limit over a certain critical volume beyond the notch tip, in order for failure to occur, the size of which correlates to the El Haddad parameter a_0 [5]. It is of some importance to underline that the critical distance method, though known and spread in the formulation by Taylor, was previously developed by Tanaka in its point and line version [6]. Taylor in [7] tried to link the critical distance approach to the mechanism of growth and arrest of mechanically short crack at notches (non-propagating cracks), worked well in case of crack-like (sharp) notches, whereas in case of short and blunt notches the predictions were poor.

The existence of non-propagating cracks at the notch root was already found by Frost in the fifties during fatigue tests on cylindrical and plate notched specimens (see for instance [8]). A major work on the prediction of non-propagating cracks at notches has been published by El Haddad et al. [5], who proposed a model which is able to predict fairly well the length of non-propagating cracks at notches, as well as the dependence of the fatigue limit on crack length. Furthermore they demonstrated the effect of the notch geometry on the initiation and propagation of small cracks. A further important contribution for the understanding of the influence of notch geometry on fatigue strength has been presented by Lukáš et al. [9]. The authors proposed an equation for assessing the critical size of non-damaging notches depending on the tensile strength of steels.

An alternative methodology based on fracture mechanics for the prediction of fatigue limits and corresponding non-propagating cracks is the so-called cyclic R-curve method. The cyclic R-curve represents the resistance force of a fatigue crack to advance, which is described in terms of a threshold stress intensity factor range ΔK_{th} depending on the crack advance Δa. To the best knowledge of the authors of this paper, the first work using

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Nomenclature

a_i	initial crack length	σ_a	applied stress amplitude
a_R	constant in the definition of the roughness-induced fatigue factor	σ_{e0}	fatigue limit for $R = -1$ (stress amplitude)
a_0	El Haddad parameter	σ_{ref}	reference stress
a^*	material constant in Peterson's formula	σ_Y	yield strength
b	exponent in the cyclic R-curve law	σ_0	reference yield stress
$f(L_r)$	ligament yielding correction function	Δa	crack advance
x	coordinate from notch root	Δa_{LC}	crack advance at the transition to the long crack regime
A	parameter in the cyclic R-curve law	ΔK	stress intensity factor range
A_5	permanent elongation at fracture	ΔK_p	plasticity-corrected stress intensity factor range
K_f	notch fatigue factor	ΔK_{th}	threshold stress intensity factor range
K_t	theoretical stress concentration factor	$\Delta K_{th,eff}$	intrinsic fatigue crack propagation threshold
L_r	ligament yielding parameter	$\Delta K_{th,LC}$	fatigue crack propagation threshold for long cracks
R	stress ratio	$\Delta K_{th,op}$	extrinsic component of ΔK_{th}
R_{eL}	lower yield stress	ΔL_r	ligament yielding parameter range
R_m	ultimate tensile strength	$\Delta\sigma_a$	applied stress range
$R_{m,\min}$	constant in the definition of the roughness-induced fatigue factor	$\Delta\bar{\sigma}_{an}$	stress range averaged over $2a_0$ from the notch
R_z	surface roughness parameter	$\Delta\sigma_e$	fatigue limit range
W	semi-width of the plate	$\Delta\sigma_{en}$	fatigue limit range for the notched specimen
ρ	notch-tip radius	$\Delta\sigma_{eR}$	fatigue limit range in presence of surface roughness
ρ^*	material constant in Neuber's formula	$\Delta\sigma_{e0}$	fatigue limit range for $R = -1$
$\sigma(x)$	local stress distribution normal to the crack plane	$\Delta\sigma_{ref}$	reference stress range

this kind of approach has been published by Yates et al. [10], who approximated the cyclic R-curve by means of the well-known Kitagawa–Takahashi diagram and use it for predicting the occurrence and length of non-propagating cracks. Sometime later Tanaka et al. [11], not only estimated the length of the non-propagating cracks, but provided also a method to evaluate their propagation threshold. A similar approach has been proposed by Akiniwa et al. [12], in which for the first time the development of the plasticity-induced crack closure has been used to explain the arrest condition of nucleated cracks. An application of the cyclic R-curve method can be traced also in the work of Taberning et al. [13].

The present work is aimed to address the influence of the surface roughness on the prediction of the fatigue limit for two different steel grades, a medium and a high strength steel. To this purpose, a procedure previously developed by the authors has been employed [14] and modified in order to deal with notches. The calculations are based on the cyclic R-curve method, in which the crack driving force has been calculated according to elastic–plastic fracture mechanics in order to deal with mechanically short cracks.

An overview on the cyclic R-curve method and the developed procedure is given in the following, then the model is applied to a simple infinite plate with different circular holes under tensile loading. Finally the calculations in case of specimens with different roughness profiles are presented and the results compared with simple approximation formulas given in the literature.

2. Material properties

2.1. Basic properties

Two different kind of structural steels have been investigated, namely the S355NL and the S960QL, both have been delivered as hot-rolled plates with 10 mm thickness. The S355NL is a medium-strength steel which undergoes a normalizing heat treatment, i.e. it is reheated above the austenite recrystallization temperature and then cooled in air. The final microstructure is characterized by a fine one-way striping in the rolling direction with fine perlite and ferrite bands. The S960QL is a high-strength steel which is delivered in a quenched and tempered condition, i.e. the material is reheated above the austenite recrystallization temperature followed by water cooling. The resulting microstructure is then fine martensite. The basic mechanical properties are summarized in Table 1. It is of some importance to see that both materials do not exhibit any anisotropic behavior in the hot-rolled condition, the response in the longitudinal (rolling direction) and transverse direction is identical.

No experimental tests have been performed in order to derive the fatigue limits. Nevertheless good and reliable estimates for smooth specimens made of steel in tension–compression tests at stress ratio $R = -1$ can be provided employing common engineering formulas like $\sigma_{e0} = 0.4 \div 0.5R_m$ [15]. In particular the fatigue limits have been estimated as follows: (i) for the steel grade S355NL $\sigma_{e0} = 0.45R_m \approx 250$ MPa; (ii) for the steel grade S960QL $\sigma_{e0} = 0.45R_m \approx 460$ MPa. Note that the values agree well with those which can be found in the literature [16–18].

2.2. Experimental derivation of the cyclic R-curve

The experimental determination of the cyclic R-curve represents one of the major issues of this work, as it is the fundamental tool for the description of the resistance force for a fatigue crack to propagate and eventually arrest, thus becoming non-propagating. The cyclic R-curve is meant to describe the evolution of the crack propagation threshold ΔK_{th} with crack advance Δa. In general,

Table 1
Basic mechanical properties. (*source:* P. Kucharczyk, RWTH Aachen.)

	R_{eL} (MPa)	R_m (MPa)	A_5 (%)
S355NL, longitudinal	371	547	32.3
S355NL, transverse	373	549	32.6
S960QL, longitudinal	968	1016	16.5
S960QL, transverse	970	1016	15.2