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Effect of cyclic plastic strain on fatigue crack growth

Michael Vormwald *

Materials Mechanics Group – Technische Universität Darmstadt, Franziska-Braun-Str. 3, D-642987 Darmstadt, Germany

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ABSTRACT

Because fatigue life calculations, which are based on the stress intensity factor, are restricted to certain limited applications, one must develop substitute procedures in more general settings. Various proposals for a crack driving force parameter in elastic–plastic fracture mechanics will be discussed. A preference in favour of the cyclic ΔJ-integral is elaborated, keeping its theoretical limitations in mind. Crack closure is also an important issue under large-scale yielding conditions. Experience cannot be extrapolated from the small-scale to the large-scale cyclic yielding regime or vice versa. The consequences for fatigue lives under variable amplitude and multiaxial loading are discussed.

1. Introduction

The fatigue crack growth rate is determined by stresses and deformations at the crack front. According to the linear theory of elasticity, a singularity appears at crack fronts. In a first-order approximation crack growth rates are usually linked to the range of the corresponding stress intensity factor. However, plastic and disruptive processes at the crack tip are responsible for crack propagation. The applicability of a parameter derived from a linear theory to describe highly non-linear phenomena is limited. Various observable phenomena of fatigue crack growth cannot be explained without considering cyclic plasticity. The influence of mean stress on the fatigue crack growth rate is a well known example where plasticity-induced crack closure supplies an explanation. The crack closure argument also provides an explanation for the various phenomena of load history effects, which may accelerate or decelerate the growth rate depending on a variety of conditions. Experience concerning fatigue crack closure cannot be transferred from the small-scale to the large-scale cyclic yielding regime or vice versa. Multiaxial and mixed mode aspects in the large-scale cyclic yielding regime are a final topic of the present paper.

2. Crack driving force parameters

The overview on the development of crack driving force parameters has been well documented by McClung et al. [1]. A first obvious attempt to move from small to large scale yielding was to replace stress by strain quantities, see for example Boettner et al. [2], McEvily [3], or El-Haddad et al. [4]. At first, only the plastic strain range replaced the stress range in stress intensity factor formulas of a given geometry. Later the total strain range was used.

$$\Delta K_\varepsilon = (\Delta\varepsilon_{el} + \Delta\varepsilon_{pl})E\sqrt{\pi a}Y \quad (1)$$

This makes more sense because a large scale yielding parameter should smoothly approach the small scale yielding parameter for vanishing plastic deformations. However, a strain-based intensity factor ΔK_ε does not provide a measure of the strain singularity at the crack front. The most important advantage of this method is its pre-eminent ease of application. If applied as suggested, not even the growth rate constants have to be re-determined. They can be directly used as determined in the small scale yielding formulation and expressed in terms of the stress intensity factor.

The theoretical shortcomings of ΔK_ε can be overcome by using the cyclic crack tip opening displacement $\Delta\delta_t$ as the crack driving force parameter. This was proposed by many researchers, see for example McEvily et al. [5], Tomkins [6], or Tanaka et al. [7]. This measure of cyclic deformation is taken as closely as possible to the location of material separation. It is generally assumed to provide a sound and unique correlation with the growth rate. However, determining it is a difficult task. McClung et al. [1] have already emphasised that knowledge of a proper driving force is of little value unless the driving force can either be calculated exactly with reasonable effort or estimated with sufficient accuracy. Simple extensions of the Dugdale [8] model led to approximation formulas, see Vormwald and Seeger [9]. An alternative way to

* Tel.: +49 6151163645; fax: +49 6151163038.
E-mail address: vormwald@wm.tu-darmstadt.de

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Nomenclature

a	crack length	N	number of cycles to failure
D	damage sum	R	stress ratio
E	Young's modulus	t	stress vector
i	counter	u	displacement vector
J	J-integral	W	strain energy density
K	stress intensity factor	x	coordinate
K'	cyclic hardening coefficient	Y	geometry influence function
m	exponent in crack growth law	δ_t	crack tip opening displacement
m_δ	constant	ε	strain
n	counter for the number of applied load cycles	σ	stress
n'	cyclic hardening exponent		

determine the cyclic crack tip opening displacement is to apply the strip yield model, Dill and Saff [10], Führing and Seeger [11], Newman [12]. The strip yield model is usually applied only to calculate the crack opening stresses. Schlitzer et al. [13] calculated fatigue crack growth based on the cyclic crack tip opening displacement extracted from strip yield model calculations. When going from large to small scale yielding conditions the parameter should asymptotically approach a bijective functional relationship with the stress intensity factor. For the crack tip opening displacement this function is provided by

$$\Delta K_\delta = \sqrt{m_\delta E \Delta\sigma_Y \Delta\delta_t} \tag{2}$$

where m_δ = 1 for plane stress conditions and $m_\delta \approx 2$ for plane strain.

The extension of the J-integral for application with cyclic loading according to Dowling and Begley [14], and Dowling [15] has by far drawn the greatest attention of researchers and engineers who want to model fatigue crack growth in the large scale yielding regime. It has also been the subject of extreme academic controversy. The definition of the cyclic ΔJ-integral is given by

$$\Delta J = J(\Delta\sigma_{ij}, \Delta\varepsilon_{ij}, \Delta t_i, \Delta u_i) = \int_\Gamma \left(\Delta W dx_2 - \Delta t_i \frac{\partial(\Delta u_{i,})}{\partial x_1} ds \right) \tag{3}$$

with

$$\Delta W = W(\Delta\sigma_{ij}, \Delta\varepsilon_{ij}) = \int_0^{\Delta\varepsilon_{ij}(\Delta\tilde{\sigma}_{ij})} \Delta\tilde{\sigma}_{ij} d(\Delta\tilde{\varepsilon}_{ij}). \tag{4}$$

The prefix "Δ" of the variables for stress σ_{ij}, strain ε_{ij}, traction t_i and displacement u_i, designates the changes in these quantities. These changes must be evaluated from a reference state. This reference state of all field variables serves as a new origin for defining increment-field variables, the latter designated with the prefix. The stress and displacement state at the time of a load reversal is a natural and sound reference state. Then the increments $\Delta(\cdots)$ designate the changes from the respective reference values. At the instant of the next reversal the field variables with a prefix are identical with the conventional ranges. However, the "Δ"-prefix does not represent changes in J and W; instead ΔJ and ΔW are functions of their arguments as defined in Eqs. (3) and (4). Wüthrich [16] proposed that a new variable, Z for ΔJ, should be used. The proposal was not widely accepted. McClung et al. [1] suggested referring to a "ΔJ-integral" rather than the "range of the J-integral"; the latter is, strictly speaking, wrong.

The path independence is the outstanding property of the J-integral and the ΔJ-integral. A path independent value is a measure of stresses and strains very near the crack front responsible for material separation processes, on the one hand, but, on the other hand, it can also be determined by far field values, uncontaminated by numerical deficiencies. The most frequent objections raised to the ΔJ-integral argued that since the J-integral was based on the theory of non-linear elasticity or (with limitation) deformation plasticity, it does not allow unloading. However, Wüthrich [16] proved the path independence if the ΔJ-integral is defined as given above.

The use of the ΔJ-integral has some remaining theoretical limitations. A first type of limitation has to do with the material's stress–strain behaviour. Yoon and Saxena [17] have pointed out that path independence is violated if the material is not completely cyclically stabilized. Also, strict compliance with path independence conditions cannot be achieved in the presence of temperature gradients and material behaviour dependent on temperature. Some remedies have been proposed, e.g. Blackburn [18], Kishimoto et al. [19], Atluri et al. [20]. These proposals for removing mathematical restrictions have recently found their way into engineering applications, see Bauerbach et al. [21].

A second type of limitation has to do with crack closure. There should be no stresses at the crack flanks; otherwise path independence is violated. The instant of a complete loss of contact would be a natural reference state. This state is recommended by McClung et al. [1]. However, even at this instant, the material at the various locations in the structure is at different stress–strain-positions on the ascending hysteresis branch. There will be no path independence during further loading. A valid ΔJ-integral can be calculated for a reversal from maximum load to crack closure load. This opens a route to deal with crack closure in connection with a ΔJ-based fatigue crack growth calculation. Choosing the instant of the maximum load as reference state, path independence is maintained during the descending reversal until crack face contact first occurs. Calculating the ΔJ-integral at the instant of first contact will provide a path independent effective ΔJ-integral, ΔJ_{eff}.

3. Crack closure under large-scale yielding conditions

Measurements of the crack opening and crack closure levels under large-scale cyclic yielding conditions have been performed by many researchers, Dowling and Iyyer [22], Rie and Schubert [23], McClung and Sehitoglu [24,25], Vormwald and Seeger [9,26], DuQuesnay et al. [27], El-Zeghayar et al. [28,29], Pippan et al. [30]. Some results for the constructional steel S460 [9] are shown in Fig. 1. Short fatigue cracks were initiated by strain controlled fatigue loading of conventional cylindrical material specimens used for low-cycle fatigue testing. The deformations in the neighbourhood of the crack were measured using a strain microstrain-gauge with a grid length of 0.6 mm. While the crack is closed, its flanks can transfer stresses by contact. This leads to a nearly homogeneous uniaxial stress state. Local near-crack strains do not differ from global strains used in the control loop of the testing machine. Upon crack opening, the micro strain gauge gets into the stress and strain shadow of the crack. The local strains are smaller than the global strains. The deviation between the