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# On the introduction of a mean stress in kinetic damage evolution laws for fatigue

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ABSTRACT

An evolutive mean value $\breve{z}$ is defined for complex loadings. It is shown to be independent from the loading shape and it tends toward the classical mean value $z_{\rm mean} = \bar{z} = \frac{1}{2}(z_{\min} + z_{\rm Max})$ for a periodic loading. The proposed definition applies to quantities encountered in the fatigue modeling of different materials: mean stress or mean hydrostatic stress for metals, mean damage driving force or mean equivalent strain for quasi-brittle materials or for composites. It gives the possibility to introduce the adequate mean stress effect in kinetic (rate) damage evolution laws. This point is illustrated for woven interlock composites in two steps, (i) by proposing an original modeling of the asymptotic Haigh diagram and (ii) by the description of full mean stress effect from kinetic damage evolution laws. The concept of evolutive mean stress gives the possibility to model fatigue under complex loading with no need to define a cycle. It applies to random fatigue as shown in different examples.



## 1. Introduction

Fatigue damage is usually modeled by means of cyclic damage laws [1,2], *i.e.* laws for damage increment per cycle $\delta D/\delta N = G(\sigma_a, \ldots)$ expressed in terms of stress amplitude $\sigma_a = \Delta\sigma/2$ and of either the mean stress $\bar{\sigma} = \frac{1}{2}(\sigma_{\min} + \sigma_{\rm Max})$ or the stress ratio $R = \sigma_{\min}/\sigma_{\rm Max}$, quantities defined over a cycle. Such a fatigue approach has only recently been applied to composites [3–6]. On the contrary, kinetic (rate) damage evolution laws relate the damage rate $\dot{D}$ to current values of the stress (or strain) tensor, of the accumulated plastic strain rate for low cycle fatigue of metals [7–9]. The mean stress effect described by such kinetic damage laws is gained from the time integration of the damage evolution law over one periodic cycle, defining the damage increment per cycle as the integral

$$\frac{\delta D}{\delta N} = \int_{1\text{cycle}} \dot{D}\, dt \tag{1}$$

Eq. (1) defines a cyclic damage law, possibly 3D, it is function of the minimum and maximum mechanical quantities over the considered cycle. It includes a mean stress effect but this effect cannot be chosen nor easily changed in order to properly fit experimental results. For instance [10] it is rather difficult to enforce the Goodman–Söderberg linear dependency $\sigma_\ell = \sigma_f^\infty(1 - b\bar{\sigma})$ of the fatigue limit (in stress amplitude) with respect to the mean stress, as encountered in fatigue of metallic materials [7,11]. It is even more difficult to gain from time integration (1) the 3D Sines extension of a linear dependency with the mean value of the hydrostatic stress $\bar{\sigma}_H = \frac{1}{2}(\sigma_{H\min} + \sigma_{H\rm Max})$

$$\sigma_\ell = \sigma_f^\infty(1 - 3b\bar{\sigma}_H) \qquad \sigma_H = \frac{1}{3}\mathrm{tr}\boldsymbol{\sigma} = \frac{1}{3}\sigma_{kk} \tag{2}$$

defining for a cyclic loading the fatigue limit in terms of octahedral equivalent stress

$$A_{\rm II} = \frac{1}{2}(\boldsymbol{\sigma}_{\rm Max} - \boldsymbol{\sigma}_{\min})_{eq} = \frac{1}{2}\sqrt{\frac{3}{2}(\boldsymbol{\sigma}'_{\rm Max} - \boldsymbol{\sigma}'_{\min}) : (\boldsymbol{\sigma}'_{\rm Max} - \boldsymbol{\sigma}'_{\min})} \tag{3}$$

with $(.)' = (.) - \frac{1}{3}\mathrm{tr}(.)\,\mathbf{1}$ the deviatoric part.

Kinetic damage evolution laws have been initially introduced for ductile failure [12,13,7]. They have later been used for the modeling of fatigue of quasi-brittle materials [39,14,15], of elastomers [17–19], of interfaces of laminated composites [35]. In these works the only attempt to describe the mean stress effect [15] was a quick and not fully satisfactory in comparison with the linear effect described by Aas-Jackobsen formula [16] for lightweight concrete.

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The main difficulty in the modeling of mean stress effect by means of kinetic (rate) damage evolution laws is the lack of definition of an average value for the stresses or strains, a definition valid in multiaxial but especially in case of non cyclic loading, a definition that can be used to parametrize the expression of the damage rate equation $\dot{D} = \ldots$. First attempts made to describe the mean stress effect within such a kinetic Continuum Damage Mechanics framework used the micro-defects closure effect, of micro-cracks opening and closure due to change of stress sign [10] or more recently used a first invariant dependent micro-plasticity criterion [20–22]: the corresponding theories were not general as they were related to metals plasticity. There were still a lack of flexibility in the modeling. In present work one describes first a way to improve such a two scale plasticity–damage modeling, for instance by allowing for the representation of bilinear mean stress effect of TA6V titanium alloy. A more general approach is proposed, based on the definition of a general evolutive mean value $\breve{z}(t)$ for any quantity $z$, $z$ being a stress $\sigma$, an hydrostatic stress $\sigma_H$, a thermodynamics force $Y$, an equivalent strain $\epsilon_{eq} \ldots$.

## 2. Mean stress effect from first invariant criterion function at defects scale

A multiaxial two scale damage model has been proposed by [23] for High Cycle Fatigue of metals. It is based on Lemaitre kinetic damage evolution law, of damage governed by (micro-) plasticity,

$$\dot{D} = \frac{Y^\mu}{S}\dot{p}^\mu \tag{4}$$

The damage strength $S$ is a material parameter, $p^\mu$ is the accumulated plastic strain at microscale $\mu$ (the defects scale), $Y^\mu$ is the strain energy release rate density (the thermodynamics force associated with damage), here also at microscale. It is function of the stress triaxiality $\sigma_H^\mu/\sigma_{eq}^\mu$ as

$$Y^\mu = \frac{\tilde{\sigma}_{eq}^{\mu\,2} R_\nu}{2E} \qquad R_\nu = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left\langle \frac{\sigma_H^\mu}{\sigma_{eq}^\mu}\right\rangle_+^2 \tag{5}$$

with $\tilde{\boldsymbol{\sigma}}^\mu = \boldsymbol{\sigma}^\mu/(1-D)$ the effective stress of Continuum Damage Mechanics and $(.)_{eq} = \sqrt{\frac{3}{2}(.)' : (.)'}$ von Mises norm. Quantities at microscale have a $\mu$ supperscript, they are related to the stress $\boldsymbol{\sigma}$ at the Representative Volume Element scale by means of Eshelby–Kröner scale transition law [24,25]

$$\tilde{\boldsymbol{\sigma}}^\mu = \boldsymbol{\sigma} - 2G(1-\beta_E)\boldsymbol{\epsilon}^{p\mu} \qquad \beta_E = \frac{2}{15}\frac{5-4\nu}{1-\nu} \tag{6}$$

The stress history in High Cycle Fatigue is obtained from an elastic computation ($\boldsymbol{\sigma} = \mathbb{E} : \boldsymbol{\epsilon}$, with $\mathbb{E}$ Hooke's elasticity tensor), when the microscale quantities, effective stress $\tilde{\boldsymbol{\sigma}}^\mu$, stress $\boldsymbol{\sigma}^\mu$, strain $\boldsymbol{\epsilon}^\mu$, plastic strain $\boldsymbol{\epsilon}^{p\mu}$, kinematic hardening $\mathbf{X}^\mu$ and damage $D$ result from the time integration, time step by time step, of elasto-plasticity coupled with damage constitutive equations (here at microscale): elasticity $\tilde{\boldsymbol{\sigma}}^\mu = \mathbb{E} : (\boldsymbol{\epsilon}^\mu - \boldsymbol{\epsilon}^{p\mu})$, plasticity evolution $\dot{\boldsymbol{\epsilon}}^{p\mu} = \dot{p}^\mu \frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}^{\mu\prime} - \mathbf{X}^\mu}{(\tilde{\boldsymbol{\sigma}}^\mu - \mathbf{X}^\mu)_{eq}}$, linear kinematic hardening $\dot{\mathbf{X}}^\mu = \frac{2}{3}C(1-D)\dot{\boldsymbol{\epsilon}}^{p\mu}$ and of course damage evolution (Eq. 4). The initial criterion function considered in this two scale damage model was von Mises criterion, without [23] or with [10] kinematic hardening,

$$f^\mu = f_0^\mu = (\boldsymbol{\sigma} - \mathbf{X})_{eq} - \sigma_f^\infty \tag{7}$$

The key point of such a modeling is the fact that the yield stress at microscale $\sigma_y^\mu = \sigma_f^\infty < \sigma_y$ is taken equal to the asymptotic fatigue limit (at infinite lifetime). As von Mises criterion $f_0^\mu$ is not pressure (hydrostatic stress) dependent, the asymptotic fatigue limit in stress amplitude $\sigma_\ell$ is found independent from the mean stress for such a first modeling. Haigh diagram *at infinite lifetime* is the constant value for the stress amplitude

$$\sigma_a = \sigma_\ell = \sigma_f^\infty \tag{8}$$

The interesting point here is that damage in fatigue is considered as part of the material behavior: it is mathematically handled by time integration of differential constitutive equations (including a kinetic damage evolution law). There is no need of counting cycles methods to deal with complex loading. This feature naturally allows for non-anisothermal applications [26].

### 2.1. Linear mean stress effect

A linear mean stress effect on the fatigue asymptote can be introduced in the two scale damage model by considering Drucker–Prager criterion function at microscale [20,21], with $a$ is a material parameter,

$$f^\mu = (\tilde{\boldsymbol{\sigma}}^\mu - \mathbf{X}^\mu)_{eq} + a\mathrm{tr}\tilde{\boldsymbol{\sigma}}^\mu - \sigma_f^\infty = (\tilde{\boldsymbol{\sigma}}^\mu - \mathbf{X}^\mu)_{eq} + 3a\sigma_H - \sigma_f^\infty \tag{9}$$

*i.e.* by making the fatigue criterion pressure/first invariant dependent, as proposed by many authors [27–30,11,31] for fatigue. From Eshelby–Kröner scale transition law (6) and incompressible plasticity still one has $\mathrm{tr}\,\tilde{\boldsymbol{\sigma}}^\mu = \mathrm{tr}\,\boldsymbol{\sigma} = 3\sigma_H$. The differences here with classical works are: (i) the infinite lifetime domain $f^\mu < 0$ is translated by micro-plasticity and (ii) the current values of the stresses are used (not the maximum nor mean values) and the modeling remains incremental. Micro-plasticity and damage are solution of kinetic differential equation so that there is no need of the definition of a cycle to calculate the time to crack initiation (it is the time at which $D(t) = D_c$, the critical damage). In A one shows from the two scale damage model constitutive equations that the asymptotic fatigue limit is linearly mean stress dependent as at infinite lifetime,

$$\begin{aligned}&\text{in 1D}: \sigma_a = \sigma_\ell = \sigma_f^\infty - a\overline{\sigma} \qquad \text{in 3D, proportional loading}\\ &\quad : A_{II} = \sigma_\ell = \sigma_f^\infty - a\mathrm{tr}\,\overline{\boldsymbol{\sigma}}\end{aligned} \tag{10}$$

so that Sines criterion is retrieved in 3D, under proportional loading assumption with octahedral stress $A_{II}$ equal to von Mises norm of stress tensor amplitude. The fatigue limit in shear is obtained as $\tau_f^\infty = \sigma_f^\infty / \sqrt{3}$ for any mean shear stress $\overline{\tau}$: it is not mean stress dependent, as observed experimentally [28,7].

### 2.2. Bi-linear mean stress effect

The mean stress effect is often nonlinear. This is for instance the case for aeronautics TA6Vpq titanium alloy (Fig. 1) for which the mean stress effect is quite strong at small mean stress but weaker at high mean stress (with a lower slope in Haigh diagram stress amplitude $\sigma_a$ *vs* mean stress $\overline{\sigma}$) as it has been shown by Gomez and Bonnand et al. [32,33]. A nonlinear or at least bilinear modeling is needed if the applications range from alternated fatigue to high mean stress loading.

A bilinear mean stress effect on the fatigue asymptote can be introduced in the two scale damage model by considering a bilinear definition of the first invariant term of criterion function at microscale, as

$$\begin{aligned}&f^\mu = (\tilde{\boldsymbol{\sigma}}^\mu - \mathbf{X}^\mu)_{eq} + K^\mu(\sigma_H) - \sqrt{3}\tau_f^\infty\\ &K^\mu(\sigma_H) = \begin{cases} 3a_1\sigma_H & \text{if } \sigma_H \leqslant \frac{1}{3}\sigma_0 \\ 3a_2\sigma_H + (a_1 - a_2)\sigma_0 & \text{if } \sigma_H > \frac{1}{3}\sigma_0\end{cases}\end{aligned} \tag{11}$$

with $a_1$, $a_2$ and the mean stress domain transition stress $\sigma_0$ as material parameters and $K^\mu(\sigma_H = 0) = 0$ so that $\tau_f^\infty$ is the fatigue