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Determination of the Stress Intensity Factor of an elliptical breathing crack in a rotating shaft

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ABSTRACT

The failures due to the propagation of fatigue cracks are one of the most frequent problems in rotating machines. Those failures sometimes are catastrophic and are sufficient to provoke the loss of the complete machine with high risks for people and other equipments. When a cracked shaft rotates, the breathing mechanism appears. The crack passes from an open state to a close state with a transition in which a partial opening or closing of the crack is produced. In this work, a new general expression that gives the Stress Intensity Factor (SIF) along the crack front of an elliptical crack in a rotating shaft in terms of the crack depth ratio, the crack aspect ratio, the relative position on the front and the angle of rotation has been developed for linear elastic materials. By the moment, no expressions of the SIF in term of these variables have been found in the literature. To this end, a quasi-static 3D numerical model of a cracked shaft with straight and elliptical cracks subjected to rotary bending using the Finite Element Method (FEM) has been made. To simulate the rotation of the shaft, different angular positions have been considered. The SIF in mode *I* along the crack front has been calculated for each angular position of the cracked shaft and for different crack geometries. The expression results have been compared with solutions obtained from the literature. It has been found that they are in good agreement. The model has been applied to other crack geometries with good results. The obtained SIF expression allows studying the dynamic behavior of cracked shafts and can be used to analyze the crack propagation.

1. Introduction

Most operation failures of rotating machines such as turbines, blades, rotors and compressors are related with the presence and propagation of fatigue cracks in one of its main components, the shafts. These elements work in rotation and are subjected to torsion and bending which produce cyclic stresses, due to the alternating tensile and compression stresses occurring for each revolution of the shaft. Under cyclic loading conditions, cracks can develop at the surface and grow across the section. Cracks limit the safety of shafts, because they can provoke their catastrophic failure and produce personal injuries and economic problems. Therefore, it is necessary to develop appropriate maintenance plans, so that the structural integrity is guaranteed with a minimum cost.

In the dynamic behavior of cracked shafts, it must be taken into account the state of the crack during the shaft rotation. Some authors have considered that the crack is always open [1,2]. However, this is an unrealistic simplification, which leads to conservative results, but does not reflect the reality. Ideally, when a shaft rotates, the crack opens and closes once per revolution. The opening and closing of the crack has been modeled in different ways. One of them is the switching crack model, in which partial opening and closing of the crack is not taken into account, the crack is assumed to be either fully open or fully closed [3–8] and the stiffness of the shaft passes from the maximum value corresponding to the closed crack (uncracked shaft stiffness) to the minimum value corresponding to the fully open crack. This model has been used quite often due to its simplicity. Another possible model is the breathing crack one, in which the amount of the opening of the crack continuously changes with the shaft rotation. The crack gradually opens and closes, depending on if it is subjected to tensile or compression stresses. In this way, the model considers partial opening and closing of the crack during shaft rotation and the stiffness variation of the shaft is gradual. The breathing crack model, has been studied, numerically and analytically, by different authors [9–19].

On the other hand, the opening and closing of the crack is very much related with a parameter denominated Stress Intensity Factor (SIF) that characterizes the stresses at the crack front.

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Nomenclature

E	Young's modulus
ν	Poisson ratio
ρ	density
L	shaft length
D	diameter
a	crack depth
α	nondimensional crack depth
β	nondimensional crack shape
γ	relative position on the crack front
θ	rotation angle
σ	maximum bending stress
K_I	Stress Intensity Factor in mode I
F_I	nondimensional SIF
CTOD	crack tip opening displacement
u_z	displacements in the z direction
θ_{c1}	rotation angle in which the crack front starts to close
N_{ij}	coefficients of the fitting of θ_{c1}
i	polynomial grade in α
j	polynomial grade in β
k	polynomial grade in γ
$\theta_{c\gamma}$	first rotation angle, for a given location on the front γ, in which F_I is null
P_{ijk}	coefficients of the fitting of $\theta_{c\gamma}$
$\theta_{2\gamma}$	first rotation angle, for a given location on the front $-\gamma$, in which F_I is null
$\theta_{o\gamma}$	last rotation angle, for a given location on the front γ, in which F_I is null
θ_{o1}	rotation angle in which the crack front is fully open
θ_{ct1}	first rotation angle in which the crack front is fully closed
θ_{ct2}	last rotation angle in which the crack front is fully closed
F_0	F_I value for the rotation angle $\theta = 0$ for a given location on the front γ
C_{ijk}	coefficients of the fitting of F_0
F_{c1}^{γ}	F_I value for the rotation angle $\theta = \theta_{c1}$ for a given location on the front γ
M_{ijk}	coefficients of the fitting of F_{c1}^{γ}
F_{o1}^{γ}	F_I value for the rotation angle $\theta = \theta_{o1}$ for a given location on the front γ
F_I^{FO}	nondimensional SIF for the fully open crack
F_I^{PO}	nondimensional SIF for the partially open crack
$F_{I,FEM}$	numerical value of the F_I
$F_{I,MOD}$	value of the F_I given by the expression

When the crack is open, the values of the SIF are positive which indicates a tensile stress field, whereas, when the crack is closed, the values of the SIF are not positive and indicating a compressive stress field. Many SIF solutions for surface cracks in round bars have been obtained by using several methods: numerical [20–30], analytical [31] and experimental techniques [32,31,33–35]. Moreover, many expressions which allow us to determine the SIF in round bars have been proposed [21,22,24–28]. Some researchers obtained SIF expressions that depend on the relative crack depth and the crack aspect ratio limited to certain points on the crack front, for example to the points located at the crack center or at the ends. In this regard, Astiz [22] calculated SIF by the Energy Release Rate using the virtual crack extension method, and derived a polynomial fitting on the numerical results using the least squares method to obtain a SIF expression at the crack center. Shih and Chen [24] developed a expression to obtain SIF values at the crack center and at the crack end from the numerical results obtained by Carpinteri [23,36]. Couroneau and Royer [25] calculated the SIF at the crack center based on the displacements in the vicinity of the crack tip and used a least-square method to determine the SIF. Other researchers obtained SIF expressions that depend on the relative crack depth, the crack aspect ratio and also take into account the relative position for the whole crack front. For example, Levan and Royer [26] calculated the SIF in round bars with transverse circular cracks using the boundary integral equation method and obtained a polynomial expression of the SIF with the numerical results using the least squares method. Shin and Cai [28] employed a finite element analysis to evaluate the SIF along the front of an elliptical surface crack in a cylindrical rod under tension and pure bending and developed a expression from the numerical results.

In rotating cracked shafts, it must be taken into account the breathing mechanism. When the shaft rotates, the crack gradually opens and closes, therefore the behavior of the shaft becomes non linear [14–16,12,18,19]. In this regard, some researchers have studied the SIF along the crack considering the opening and closing of the crack [37,38], but without consider the non linear behavior. For example, Carpinteri [37] calculated through a three dimensional finite element analysis the SIF along the crack front for two rotation angles (0° and 90°) and obtained a general SIF expression for any rotation angle that can be expressed as a linear combination of sines and cosines.

For the first time, in the present paper, a general expression to determine the SIF along the crack front during a shaft rotation in terms of the crack depth ratio, crack aspect ratio, relative position on the front and the angle of rotation for linear elastic materials has been developed. At present, no expressions of this kind have been found in the literature, in the knowledge of the authors, hence its determination can be very useful to study the dynamic behavior of cracked shafts. To this end, a numerical study using the commercial finite element code ABAQUS [39] has been carried out. A quasi-static 3D cracked shaft model has been developed in order to determine the SIF along the crack front for each angle of rotation and for different crack geometries. The work allows knowing, through the study of the SIF, the opening/closing state of the crack.

2. Numerical model

2.1. Cracked shaft model

In the present work, a 3D quasi-static numerical study, through the Commercial Code ABAQUS [39], has been carried out to obtain the SIF along the crack front of a shaft subjected to rotary bending. In Fig. 1 a detail of the model with an open crack is shown.

The shaft total length is $L = 900$ mm, whereas the diameter is $D = 20$ mm. The material of the shaft is aluminum with the

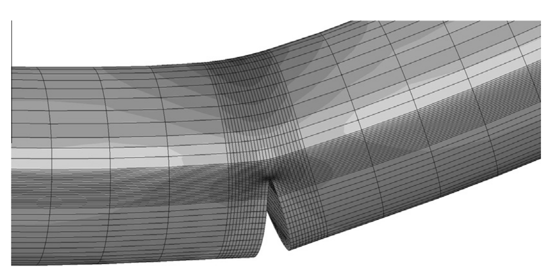

Fig. 1. Detail of the model of the shaft.