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Effects of crystallographic orientation and grain morphology on crack tip stress state and plasticity

M.E. Kartal[a], M.A. Cuddihy[b,*], F.P.E. Dunne[b]

[a] Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK
[b] Department of Materials, Imperial College London, London SW7 2AZ, UK

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ABSTRACT

The Sih, Paris and Irwin analytical solution for cracks in anisotropic elastic media has been developed for an hcp Ti single crystal and shown to lead to crack tip normal stresses which are independent of crystal orientation but other stress components which are dependent. Detailed finite element studies confirm that the stress intensity remains independent of crystal orientation but ceases to do so in an edge-cracked bi-crystal.

The incorporation of crystallographic slip demonstrates that single-crystal crack tip stresses largely remain independent of crystal orientation but that the plastic zone size and shape depends greatly upon it. Significant differences result in both the magnitude and extent of the plasticity at the crack tip with crystallographic orientation which can be quite different to that predicted using Mises plasticity. For an edge crack terminating in a bi-crystal, the slip fields which result depend upon both crystal mis-orientation and morphology.

1. Introduction

Since the 1950s, titanium has been the mainstay for critical rotating components such as discs and blades, in the low to high pressure sections of the compressor of gas turbine engines [1]. Titanium alloys offer several key advantages including reduced density, high fracture toughness, high strength and corrosion resistance. Indeed, the latter two qualities have led to extensive titanium use in biomedical devices [2]. Particular alloys may be used throughout aircraft production, such as landing gear, nacelles and fuselage [3]. The 'workhorse' of the aerospace industry is the alloy Ti–6Al–4V which in some instances may make up 80–90% of the titanium used on an aircraft [3], and accounts for approximately 56% of the market share of all titanium production [2]. Commercially pure (CP) titanium only accounts for 26%, with the remainder largely occupied by various alpha and alpha–beta alloys.

Cold dwell fatigue [1,4–10] refers to a failure mode observed in titanium alloy components due to a stress hold (dwell) at peak stress during cyclic loading at ambient temperatures. This issue, first recognised in the early 1970s after the uncontained failure of two titanium alloy aero-engine fan discs [1], remains a serious concern for all engine manufacturers [4].

Fig. 1 shows the results of a fatigue performance study on disc samples of the titanium alloy IMI834 [5] in which the effect on lifespan of the two minute dwell may be clearly seen. Research efforts have been focused on producing a quantitative predictive relationship between structure, texture and properties, but have not been completely successful. Aero-engine manufacturers such as Rolls-Royce consequently rely on expensive and time consuming component tests which attempt to establish empirical relations that may be incorporated in the design methodologies [6].

The mechanistic origin of the cold dwell debit is argued to be fatigue facets which are micro cracks occurring in areas of near-uniform crystallographic orientation in titanium alloys [12] and have been observed in experiment many times [5,11–13]. Fig. 2, taken from an experiment conducted by Sinha et al. [13], shows a facet fracture surface on a sample of Ti-6242 (or Ti–6Al–2Sn–4Zr–2Mo). This dwell-fatigue test included a load control of a peak stress of 869 MPa and included a dwell of two minutes at this peak stress. The dwell fatigue specimen failed in 447 cycles whereas the normal, cyclically loaded, fatigue specimen failed after 24,000 cycles; a dwell-debit of 54. These two specimens had the same specimen geometry and were machined from the same forging. It is noted by Sinha et al. [13] that while the precise values for lifespan reduction (dwell-debit) vary, they are consistently large.

Cold dwell facets are thought to develop progressively (as opposed to instantaneously) but nonetheless very much more rapidly than under conventional fatigue regimes. Once nucleated, in a region usually characterised by uniform basal crystallographic orientation, they are sometimes found to remain unchanged during

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* Corresponding author.
E-mail address: m.cuddihy12@imperial.ac.uk (M.A. Cuddihy).

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Nomenclature

σ_{ij}	stress in the direction indicated by the subscript, x–y Cartesian and $r-\theta$ for polar
ε_{ij}	strain in the direction indicated by the subscript, x–y Cartesian and $r-\theta$ for polar
$\varphi_{1,2}$	Euler angle
s_{ij}	element of the compliance matrix
μ_j	roots of the characteristic equation, $j = 1, 2, 3, 4$
K_j	stress intensity factor, $j = I$ or II depending on mode
$\boldsymbol{R}$	rotation matrix
$\boldsymbol{T}$	transformation matrix
$\boldsymbol{S}$	rotated compliance matrix
a, b	crack length and plate width, respectively

subsequent cyclic loading, but other times to propagate into adjacent grains.

Early pioneering work by Rice et al. in the 1980s [14–17] developed and applied asymptotic analyses to solve for crack tip fields in elastic-ideally plastic fcc and bcc single crystals; this approach was extended to hcp and layered materials by Gupta [18]. Cuitiño and Ortiz [19] investigated three dimensional crack tip fields in single crystal copper specimens under four-point bending test conditions; their numerical results agreed with that of the earlier analytical studies of Rice et al. The present work builds upon the analytical HRR-fields based research of Rice et al. by utilising the full computational power of crystal plasticity finite element methods.

The first to address crack nucleation in a rigorous manner was Stroh [20]. Later studies have addressed facet nucleation criteria (e.g. Kirane and Ghosh [21], and Dunne and Rugg [22]) and in particular, hypothesised the role of normal and shear stress relative to a basal plane in nucleation. However, the mechanics of nucleated facets, either partially developed across a region of uniform crystallographic orientation, or subsequently propagating into adjacent grains, has not received attention. Particularly, the stresses and stress intensities and localised (plastic) slip at the crack tip generated by a facet, and influenced by local crystallographic details of morphology and orientation, leading to very strong elastic and plastic anisotropy, have not been addressed.

This paper therefore presents a fundamental assessment of stress and stress intensity generated by an existing facet within, initially, a single crystal in which the crystallographic orientation is varied with respect to remote loading direction. The work is extended to consider the presence of a facet within bi-crystals with specified and varied crystallographic orientations and sizes in order to investigate the roles of grain constraint and crystallographic orientation in stress and stress intensity local to the crack tip. Finally, the anisotropic nature of the slip developed local to the facet tip is investigated with respect to crystallographic orientation.

In the next section, the fundamental analysis of stress intensities in anisotropic media are addressed, which are specialised for hexagonal close packed (hcp) elastic anisotropy. An analytical analysis is presented of the stresses local to an edge crack in an elastically anisotropic hcp single crystal and their dependence on crystallographic orientation is investigated. An edge crack in an elastically anisotropic Ti hcp bi-crystal is then addressed using a finite element model in order to investigate the role of the combination of crystallographic orientation on stress and stress intensity at the crack tip. This is followed by a brief description of the crystal plasticity slip model which is employed to investigate the accumulated slip fields ahead of the crack tip, taking full account of the anisotropic nature of the slip activity. We then address the nature of the crystal slip fields established at the crack tip, and their dependence on crystallographic orientation and relative grain size. In passing, the slip fields so determined are compared with the plastic strain fields obtained from Mises plasticity, but while maintaining the elastic anisotropy of the crystals. Finally, a quantitative analysis of accumulated slip at the crack tip as a function of crystallographic orientation is also presented.

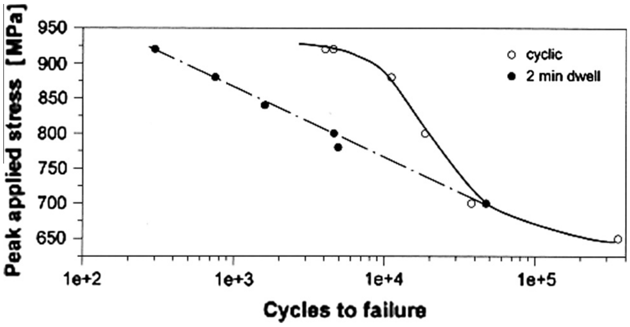

Fig. 1. S–N curves illustrating dwell-debit [5].

2. Analytical crack-tip stress fields in 2D anisotropic media

In this section, elastic anisotropy associated with hcp crystal structure is considered since this is of importance in the context of Ti alloy facet nucleation. An edge crack is first considered in a homogeneous single crystal with arbitrary crystallographic orientation with respect to the crack direction and remote loading.

2.1. Stress field formulation

Consider a plate, with its major dimensions lying in the xy plane and the z-axis directed into the page, as shown in Fig. 3(a). The x and y axes are parallel and normal to the crack surface respectively.

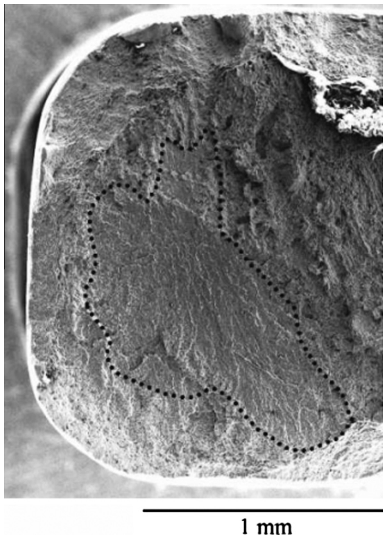

Fig. 2. Dwell-fatigue fracture surface, faceted initiation site marked by a dashed line, after [13].