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Effects of the transverse stress on biaxial fatigue crack growth predicted by plasticity-corrected stress intensity factor

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ABSTRACT

The transverse stress has an important effect on the biaxial fatigue crack behavior. However, the experimental evidence has provided conflicting indications: it is sometimes considered to increase, decrease or have no effect. These complex phenomena cannot be rationally explained by the existing mechanical models. The effect of the transverse stress on the fatigue crack growth behavior is still one of the most puzzling questions in biaxial fatigue. Physically, this effect is a transverse stress induced plasticity phenomenon. In this paper, a plasticity-corrected stress intensity factor (PC-SIF) is proposed to describe the effect of transverse stress on biaxial fatigue. By use of this new crack driving force some important phenomena associated with transverse stress are predicted. Comparisons with experimental results showed that the PC-SIF as an effective mechanical parameter is capable of predicting the effects of the crack length, the stress level, cyclic stress ratio, biaxial stress ratio and phase difference on the biaxial fatigue crack growth. Consequently, the alleged conflicting experimental results have been rationally explained by the PC-SIF.

1. Introduction

Fatigue crack growth (FCG) under biaxial stresses has received considerable attention. The effect of the transverse stress applied parallel to a crack on the FCG behavior is the most concerned problem in the biaxial fatigue. It has been shown that the transverse stress has a significant effect on the FCG behavior [1–4]. However, the transverse stress contributes nothing to the elastic stress intensity factor (SIF). Hence, the effect of the transverse stress on the FCG behavior cannot be estimated from the classical Paris law that is on the basis of elastic SIF range. Nevertheless, the transverse stress affects the shape and size of the plastic zone occurred at the crack tip [5–7]. Thus, the effect of the transverse stress on the biaxial FCG is a plasticity-induced phenomenon. It can be described only when the crack driving force is correlated with the crack-tip plastic zone.

Several methods have been proposed to correlate the FCG with transverse stress, including models based on the plasticity-induced crack closure (PICC) [8–11], plastic zone size [12,13], strain energy theory [14,15], ΔJ based approach [16], plastic blunting at crack tip [17]. All of these methods are in some way associated with the crack-tip plastic deformation during the application of a transverse stress.

The effective SIF based on the PICC, defined as $\Delta K_{eff} = K_{max} - K_{op}$ (where K_{max} and K_{op} are the SIF calculated for the maximum load and for the crack opening load), is the most accepted concept, and has been widely used as a critical mechanism responsible for the plastic zone effect. This concept placed the emphasis on the impact of the contact stresses behind the crack tip, which are removed at the crack opening load. However, when the cracked body is at the opening load the plasticity-induced residual stresses are still present in the plastic zone due to the memory effect of the plastic deformation, which has an important effect on the crack growth behavior [2,18]. The PICC model only accounts for the effects of the contact stresses on the cracked surfaces, ignores the variation in crack-tip stress field during the crack contact. Furthermore, the PICC concept is not adequate to properly describe crack growth at load ratio $R < 0$ [19]. Some additional drawbacks associated with the PICC methodology have been discussed in detail by Dinda and Kujawsk [20], Sadananda and Vasudevan [21] and Kujawski [22].

On the other hand, the experimental evidence has provided conflicting indications: the effect of an applied static or cyclic transverse stress is sometimes considered to increase, decrease or have no effect on the FCG rate [14]. These anomalies highlighted the complex nature of the effect of the transverse stress on the crack-tip plastic zone, thereby the biaxial FCG behavior. Up to now, none of the existing models can rationally explain these complex experimental phenomena. The effect of the transverse stress on the FCG behavior is still one of the most puzzling questions in biaxial fatigue.

Recently, Li and his co-authors [23–26] demonstrated that a plastically-deformed zone can be identified with a homogenous

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inclusion of transformation strain by means of the Eshelby equivalent inclusion method. The change in crack tip SIF due to the crack-tip plastic zone can then be quantitatively evaluated by the transformation toughening theory [27]. According to this plastic zone toughening theory, the plastic zone-induced SIF is expressed as

$$\Delta K_{\mathrm{I-pl}} = \frac{1}{2\sqrt{2\pi}}\int_{\Omega} r^{-3/2}\left[\frac{K_{\mathrm{I-el}}}{\sqrt{2\pi r}}\left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+3\sin^2\theta\cos\theta\right)\right.$$
$$\left.+3(\sigma_{11}-\sigma_{22})\sin\theta\sin\frac{5\theta}{2}-6\sigma_{12}\sin\theta\cos\frac{5\theta}{2}-(\sigma_{11}+\sigma_{22})\cos\frac{3\theta}{2}\right]d\Omega, \quad (1)$$

for mode-I crack under small-scale yielding and plane strain conditions [25,28]. In Eq. (1), $K_{\mathrm{I-el}}$ is the remotely applied elastic SIF, Ω is the area of the plastic zone around crack tip, and σ_{11}, σ_{22}, σ_{12} are the stresses in the plastic zone. Thus, a plasticity-corrected (PC) SIF, $K_{\mathrm{I-pc}}$, was defined as

$$K_{\mathrm{I-pc}} = K_{\mathrm{I-el}} + \Delta K_{\mathrm{I-pl}}. \quad (2)$$

The PC-SIF was proposed as a new mechanical driving force parameter for predicting FCG rate in uniaxial fatigue. Some important phenomena associated with the plastic zone around a fatigue crack tip, such as the effects of load ratio R, overload and the FCG behavior under cyclic compression, have been well described by the PC-SIF [29–31]. Comparisons with extensive experimental data showed that the proposed PC-SIF is an effective single mechanical parameter capable of correlating the effects of crack-tip zone.

Since the effect of the transverse stress on the FCG is a typically plasticity-induced phenomenon, it should be well predicted by the PC-SIF. In order to give a rational explanation for the conflicting experimental observations and provide an analytical method for FCG rate in biaxial loading, in the present study the application of the PC-SIF to biaxial fatigue will be focused on predicting the effects of the applied static or cyclic stress level, biaxial stress ratio, phase difference, cyclic stress ratio and crack length on the FCG behavior.

2. The PC-SIF range $\Delta K_{\mathrm{I-pc}}$

For a fatigue crack, the PC-SIF range, $\Delta K_{\mathrm{I-pc}}$, is defined as [29]

$$\Delta K_{\mathrm{I-pc}} = K_{\mathrm{I-pc}}^{\max} - K_{\mathrm{I-pc}}^{\min}, \quad (3)$$

where $K_{\mathrm{I-pc}}^{\max}$ and $K_{\mathrm{I-pc}}^{\min}$ are the PC-SIF associated with the maximum and minimum load in a loading cycle according to Eq. (2). When the crack closure occurs during cyclic loading the term of $K_{\mathrm{I-pc}}^{\min}$ in Eq. (3) is calculated from the opening load, denoted by S_{OP}, i.e.,

$$K_{\mathrm{I-pc}}^{\min} = K_{\mathrm{I-el}}(S_{\mathrm{op}}) + \Delta K_{\mathrm{I-pl}}(S_{\mathrm{op}}), \quad (4)$$

where

$$\Delta K_{\mathrm{I-pl}}(S_{\mathrm{op}}) = \frac{1}{2\sqrt{2\pi}}\int_{\Omega} r^{-3/2}\left[\frac{K_{\mathrm{I-el}}(S_{\mathrm{op}})}{\sqrt{2\pi r}}\left(2\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+3\sin^2\theta\cos\theta\right)\right.$$
$$\left.+3(\sigma_{11}-\sigma_{22})\sin\theta\sin\frac{5\theta}{2}-6\sigma_{12}\sin\theta\cos\frac{5\theta}{2}-(\sigma_{11}+\sigma_{22})\cos\frac{3\theta}{2}\right]d\Omega. \quad (5)$$

Thus, the crack closure effect on the FCG is taken into account.

The memory effect of the plastic deformation requires that once a material element has been plastically deformed, it will be taken as a part of the current plastic zone even if this material element deforms elastically at present loading stage. Thus, for a growing crack the area of the plastic zone, Ω, includes the plastically deformed wake zone [29].

3. Finite element simulations

The finite element (FE) model analyzed in this study is a 100 mm × 100 mm CCT-specimen in plane-strain conditions (Fig. 1a). The geometrical details of the initial crack are shown in Fig. 1b. Due to symmetries, only one-quarter of the specimen was modeled. The FE-mesh details near the notch and crack tip are shown in Fig. 1c. The crack tip is located in a uniform mesh region with finest element size due to the high stress and strain gradients near the crack tip. Referring to the coordinates system, the displacements in the y-direction were constrained for the nodes along the $(x \geqslant a, y = 0)$ plane, and the displacements in the x-direction were constrained for the nodes along the $x = 0$ plane. Surface contact elements were used on the crack surface $(x < a, y = 0)$ to model contact behavior of the crack surfaces. The lower symmetric part of the specimen served as the master surface and the upper cracked surface was the slaved one.

The model is subjected to biaxial stresses. A uniform vertical cyclic stress σ is applied to its horizontal edges, while a uniform transverse stress τ is applied along the vertical edges. The transverse stress τ may be either static or cyclic. It is expressed by $\tau = \lambda\sigma_{\max}$ and $\tau = \lambda\sigma(\varphi)$, respectively, for static and cyclic transverse stress, where $\lambda = \sigma_{\max}/\tau_{\max}$ for $\tau_{\max} > 0$ and $\lambda = \sigma_{\max}/\tau_{\min}$ for $\tau_{\max} \leqslant 0$, defines different biaxial stress states, and φ is the phase difference between σ and τ-stresses. Depending on the λ value, the cyclic stress amplitude of the τ-stress may be different from the cyclic stress σ, but it has same load ratio $R(=\sigma_{\min}/\sigma_{\max})$. Some typical load conditions are schematically illustrated in Fig. 2. For example, the load condition of $\tau = \sigma(\pi)$ shown in Fig. 2e indicates the cyclic τ-stress having same cyclic stress amplitude and load ratio, but with 180 deg phase difference.

For the CCT-specimen the value of the elastic SIF range is expressed by

$$\Delta K_{\mathrm{el}} = \Delta\sigma\sqrt{\pi a \sec(\pi a/W)} \quad (6)$$

where $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ for $R \geqslant 0$, and $\Delta\sigma = \sigma_{\max}$ for $R < 0$. a/W is the crack length normalized by specimen width W.

The numerical analyses are performed by using the FE-code ABAQUS. For all the FE simulations where a propagating crack was considered, a crack-tip node is released when the applied stress σ reached its maximum in a loading cycle, creating an effective crack growth increment of one fine mesh unit per cycle. Thus, the fatigue crack growth rate, da/dN, depends on the mesh size in front of crack tip. At present, it is not possible to choose crack increments comparable to experimental crack growth rate per cycle (for example, ranged in $10^{-7}/10^{-4}$mm/cycle for most engineering materials), because this would results in enormous

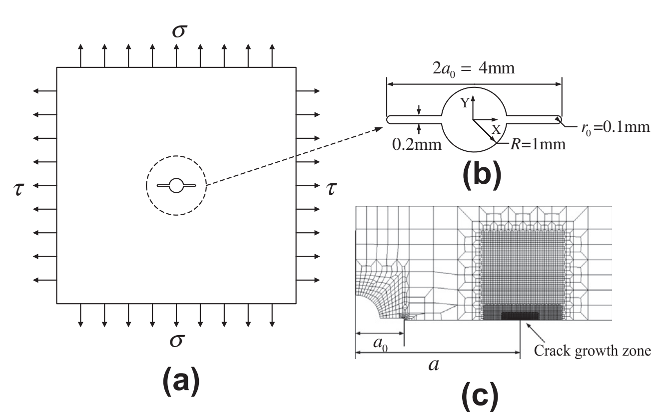

Fig. 1. The CCT-specimen model and FE-mesh details near the notch and crack tip.