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Evaluation of mechanical component fatigue behavior under random loads: *Indirect frequency domain method*

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ABSTRACT

The fatigue behavior assessment for mechanical components subjected to random loads is traditionally conducted in time domain. An alternative frequency domain procedure is applicable under the hypothesis of stationary *Gaussian* stress state. The main objectives of this work consist in the description of the limits of applicability for the frequency domain methods and in the proposal of an original procedure (*indirect method*), that combines the advantages of the dynamic analysis conducted in the frequency domain, with the fatigue direct assessment criteria in time domain. Due to the stochastic nature of the fatigue damage under random stresses, a deterministic approach cannot be adopted, however, according to the common practice, a mean value of the damage can be assessed, paying attention that the time history used is sufficiently long. This makes sure the random error is negligible as it is of the same order of magnitude of the other approximations contained in the simulation process. In this paper the authors investigate under which hypothesis this method is valid and they propose a tool for the significance test of time data with regards to their capability in reproducing the right fatigue cycle amplitude distribution of the specific random process.

1. Introduction

During the last decades a significant effort has been done in order to apply the principles of the frequency analysis to the fatigue behavior assessment of mechanical components subjected to random loads [1–3]. However the advantages of the frequency domain are not fully effective because of the problems in the translation of some damage assessment tool from the time domain to the frequency domain, as the time domain is the privileged environment where they have been developed and designed to work in. The major consequence is that the time domain approach is currently a milestone in the fatigue damage estimation of practical applications, both starting from laboratory or real measurements, conducted on physical prototypes, and from numerical simulations, conducted on virtual prototypes.

Although the difficulties are considerable, the research has been focused in the frequency domain alternative approach, as it offers the chance to work with the power spectral density function (PSD) of stress or strain process, under the hypothesis of random *Gaussian* stationary one. The aim consists in the development of procedures for the assessment of fatigue behavior that combine the most important tools developed by the scientific community in terms of simulation methodologies of the mechanical system, methodologies for the state of stress evaluation and methodologies for the estimation of the fatigue life. The overall optimized procedure would allow to quickly simulate the behavior of the whole system, without any loss of result reliability in comparison with the reference procedure. The idea comes exactly from the combination of the major advantages derived by the two domains, obtaining an innovative assessment path, here defined as *indirect approach*: the dynamic analysis of the system (multibody system MBS, finite element, FEA) performed in the frequency domain, combined with a method for the reconstruction of the stress state developed by the authors [4,5], is coupled with the classical *direct approach* used in time domain by processing the time history of the equivalent process [6] by using classical counting methods.

Thus a reliable fatigue life estimation can be achieved through a proper reconstruction of a time domain process set starting from the representation in frequency domain of the state of stress expressed in the form of PSD matrices.

The described scenario supplies different directions for the research. As a matter of fact, the fatigue life of structures subjected to loads with a significant random component represents a complex problem of stochastic nature [7]: in addition to the well-known material strength behavior, a further statistical scatter is introduced by the variability of the fatigue cycle number a certain material can resist before it fails when subjected to an assigned stress amplitude. The simplified approach, usually applied, neglects the stochastic nature of the problem by assuming that

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the available sample time history contains a fatigue cycle distribution already converged, not affected by any random error. One of the objectives of this work concerns the investigation about the effect of this approximation with respect to the total number of cycles in the reference time history. In practice a single damage value, calculated by a sample limited time history, representative of a stress process, will be coupled with the confidence limits of the estimation. This means that the fatigue life expectancy cannot be considered as a unique value, but as a probability distribution which converges to a single value as the number of cycles in the time history increases.

The concept is shown in Fig. 1: given a stationary random process $X(t)$, considering a number m (in the case of Figure $m = 50$) of its possible time history samples characterized by long duration, the fatigue damage progress can be analyzed during the different m time histories. The damage is calculated by processing each time history with the *Rainflow* cycle counting method [8] and cumulating the contribution of the various cycles by using the curve of the material fatigue strength with reliability 50%, in order to consider just the stochastic contribution of the random process and neglecting that one of the material component. Thus, the unique relationship between each sample $x_i(t)$ and its per time unit damage can be represented by a random variable D_t at each t. A sample set, of size m, for the random process $D_t(t)$ is defined, that can be adopted to estimate the damage probability distribution at any time, being valid the following assumptions:

- The distribution of D_t can be reasonably assumed *Gaussian*.
- The mean value $\mu(t)$ of the distribution does not change with t.
- The standard deviation $\sigma(t)$ of the distribution tends to zero as the time t goes to infinite.

As it will be described later, these assumptions are in general only partially verified, but the limit can be easily fixed by appropriate corrections. Under these conditions, it will be possible to estimate mean and variance of the population of the process possible samples for every time duration of the samples. It is a common practice to have one or more time histories, experimentally measured or numerically simulated: looking at Fig. 1, it can be quickly concluded that the significance of the acquired data increases as the size m of the sample, because the amplitude of the confidence interval decreases as m increases and the variance decreases. Obviously, also the significance of a single time history is strongly related to its duration. For example, considering, again, the previous example in Fig. 1, if the available data for the damage estimation were only limited to a duration of 10 s the theory of the *sample mean* [9] would have provided, for the expected value of the damage, a confidence interval of 95% $[7.5 \times 10^{-8}, 10.28 \times 10^{-8}]$, while assuming the data of a duration of 2500 s the range amplitude has been reduced to $[9.82 \times 10^{-8}, 10.09 \times 10^{-8}]$, that means the uncertainty in the estimate decreases from 37% to about 3%. The random error analysis is important in order to assess the level of uncertainty related to the total duration of the time measured (or simulated) data and it is the basis of the probabilistic approach to fatigue; however, the practical applications are more interested in the inverse problem, that is how to establish the size of the sample, once the maximum acceptable error has been fixed. The answer to this question can be considered the most important aim of this work.

However, the effort for the development of a direct method in the frequency domain is not concluded yet. It would solve many actual sources of uncertainties and make feasible the optimization of a procedure able to provide a reliable estimation at the early design stage, while probabilistic methods, such as the *indirect method*, would provide accurate estimations at the late design stage.

2. The frequency *direct method* under uniaxial stress states

Under general stress conditions, the output of a system frequency analysis performed by PSD approach by i.e. Finite Element

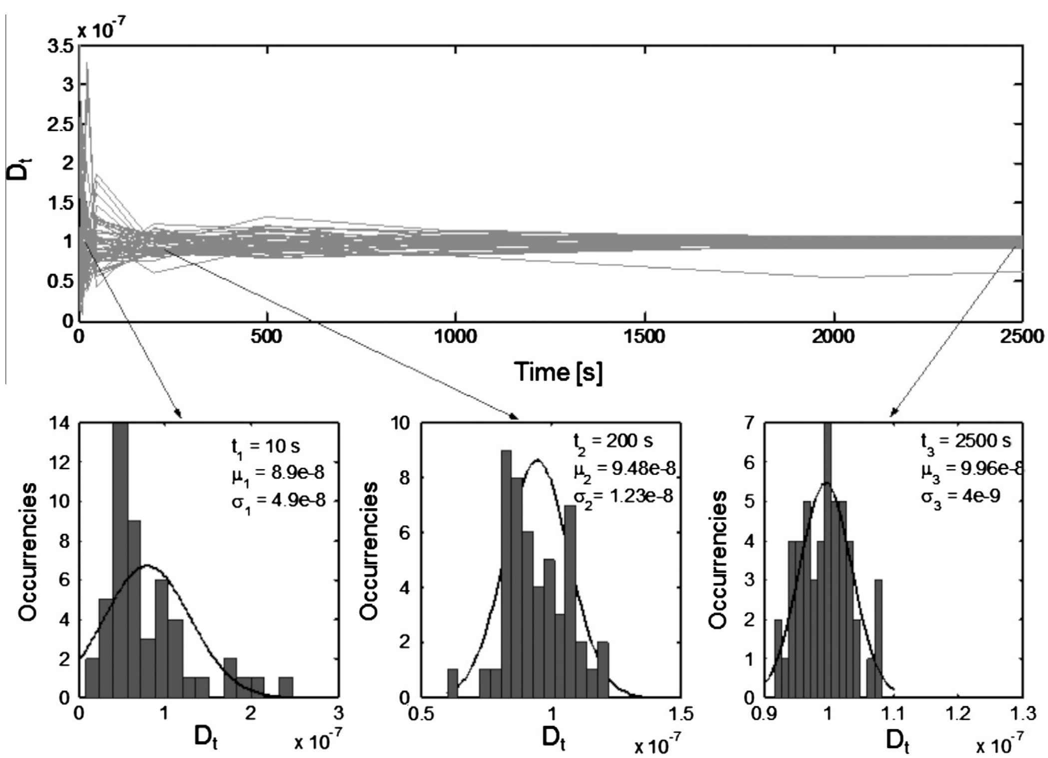

Fig. 1. Fatigue damage rate (D_t) probabilistic analysis of a stress random process.