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Formulation of gradient multiaxial fatigue criteria

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ABSTRACT

A formulation of gradient fatigue criteria is proposed in the context of multiaxial high-cycle fatigue (HCF) of metallic materials. The notable dependence of fatigue limit on some common factors not taken into account in classical fatigue criteria, is analyzed and modeled. Three interconnected factors, the size, stress gradient and loading effects, are here investigated. A new class of fatigue criteria extended from classical ones with stress gradient terms introduced not only in the normal stress but also in the shear stress components, is formulated. Such a formulation allows to capture gradient effects and related "size" effects, as well as to cover a wide range of loading mode, then can model both phenomena "Smaller is Stronger" and "Higher Gradient is Stronger". Gradient versions of some classical fatigue criteria such as Crossland and Dang Van are provided as illustrations.

1. Introduction

In recent years, there has been an increasing interest in developing fatigue criteria for metals capable of dealing with high stress gradient (around notches, voids, contacts, etc.) and particular issues related to small scales. Examples are found, on the one hand in notches and fretting problems [1–6], and on the other hand in problems related to small electronic components and electromechanical devices. At sufficiently small sizes, some factors (size, gradient and loading effects) which effects on fatigue limits are inherently not captured by classical fatigue criteria, become important and must be taken into account through new criteria. Among them, experimental evidences show three interconnected ones: size effect, gradient effect[1] and loading effect (cf. [7–14]). A visible general correlation between these factors is that, *"the smaller the size, the higher the gradient, then the higher fatigue resistance"*. There are also cases where the gradient exists but independent from the size, although both influence on material strength (e.g. residual surface stress cases). For the sake of further analyses, it requires to clarify what are the sources of the size effect by isolating it from the gradient effect. Size effect is commonly considered as the pure size effect related, in fatigue, to the metallurgical defects and heterogeneity of material. For materials presenting large defects (e.g. cast iron, cast alloys) or for very high strength steels showing surface defects introduced by the manufacturing process (machining or forging) probabilistic approaches, mostly based on the weakest link concept, are shown to be relevant tools for modeling the size effect [15–18]. Indeed, with the increase of the specimen size, the probability of finding a critical defect increases and consequently the fatigue resistance decreases. The intensity of the pure size effect is also known to depend on the microstructural heterogeneities of the material or the surface integrity of the loaded component. In [12], size and gradient effects are experimentally investigated and modeled with two statistical models based on volume or surface integration. The same fatigue limits in tension and rotating bending are found for the cast iron, showing that the influence of the flaws is greater than the effect of the gradient of the applied loading (a different observation is done for free flaw mild steel C36). Thus, for the defect material, the pure size effect may more influence the fatigue strength than the stress gradient effect. In this paper, we only consider free defect materials. In this case, size effect is proved insignificant compared to the gradient effect at the considered scale (e.g. tension–compression fatigue test in Fig. 5, [19,7]). Then a preliminary qualitative remark is that, such a pure size effect just is a part, but not enough to explain the fact well known as *"Smaller is Stronger"* that we observe in fatigue tests.

The gradient effect is another factor which may help to interpret that fact. Such effect, termed here *"Higher Gradient is Stronger"*, is roughly related to three sources: boundary condition, loading mode and size. The first is associated with constraints on dislocation glide (passivated surfaces and interfaces, boundary layers, etc.); the second concerns loading type which decides the spatial stress distribution state in the solid (null gradient in tension–compression, non-zero gradient in bending, etc.); the last is associated with the size (e.g. geometry and grain sizes). For instance, in

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[1] In the current work, this must be understood as stress gradient effect.

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bending test, the smaller the beam radius the higher the stress gradient (and the higher the fatigue limit). Experimental results [20,7] on the variation in fatigue strength at various radii conclude to the dominance of the gradient effect upon the pure size effect. Then the sources of the gradient effect prove two things: first, "Smaller is Stronger" experimentally observed is mainly attributed to the gradient effect in the cases considered here, rather than totally to the pure size effect as usually believed; second, the gradient effect, i.e. "Higher Gradient is Stronger", is really a phenomenon different from the size effect.

All previous analyses for both the size and gradient effects imply that although the size and gradient effects are intimately interconnected and usually confused in the literature, they are actually two distinct phenomena. The former only contributing in part to "Smaller is Stronger" and requiring to be modeled by other approach, is negligible compared to the latter and thus left out in the current study; whereas the latter is not only "Higher Gradient is Stronger" but also a main factor contributing to "Smaller is Stronger" that we observe, and is the object of study here. In brief, from phenomenological aspect, "Higher Gradient is Stronger" is naturally related to the gradient effect only, while "Smaller is Stronger" is related to both pure size and gradient effects where the latter is dominant. Then "Smaller is Stronger" here is just a "visible image" of gradient effect rather than the size effect from mechanical point of view. From phenomenological point of view, "Smaller is Stronger" is however an experimentally observed fact that evokes an intuitive relation to the size rather than the gradient. For this reason, henceforth in this research, the terminology *"size effect"* (placed within quotes) is still used for "Smaller is Stronger", but as an apparent size effect; and the terminology *gradient effect* is used for "Higher Gradient is Stronger". In such a sense, an important conclusion drawn is that, *taking into account only gradient effect (related to all its sources) is enough to capture both "size effect" and gradient effect on fatigue resistance*.

In this study, only cases where the gradient effect is present apart from the inherent pure size effect, are considered. As in [7], the notch effect – regarded as a particular case of the gradient effect, is left out in the study restricted to macroscopically elastic behavior or stabilized elastic shakedown state [21]. In such a context and along with the notable conclusion above, Gradient Fatigue Criteria with stress gradient terms introduced are capable to capture the "size", gradient and loading effects, and thus to model both phenomena "Smaller is Stronger" and "Higher Gradient is Stronger", as found in the applications considered here.

Classical fatigue criteria without material length scale predict no size, gradient neither loading effects. The objective is to establish a new class of fatigue criteria for considering the previous factors. Existing approaches dealing with such problems are (cf. [8–11,13]): (i) critical layer of Flavenot and Skally [22]; (ii) distance approaches such as: effective distance approach of Pluvinage [4], Qylafku et al. [5]; theory of critical distances, Taylor [2], Araujo et al. [3]; (iii) nonlocal approaches such as: maximum stressed-strained volume by Sonsino et al. [23]; energy based criterion of Palin-Luc and Lasserre [24]; volumetric energy based criterion of Banvillet et al. [9] and Palin-Luc [10]; gradient method proposed by Brand and Sutterlin [25,26]; and (iv) local approaches such as: gradient dependent criterion of Papadopoulos and Panoskaltsis [7]; that of Ngargueudedjim et al. [27], and several derivatives based on this work [7] proposed by Fouvry et al. [1,28] and Weber [13] (gradient version of the criterion of Robert [29], and that of Fogue [30,31]).

The review of Papadopoulos and Panoskaltsis [7] is re-used and developed to make more clear the connection as well as the distinction between the effects by analyzing the role of each dimension of specimen in fatigue resistance. It is shown that two issues remain: first, the non-effect of the shear stress gradient on fatigue limits is only found for some metals – but not all; second, the influence of the stress gradient amplitude must be clarified. Thereby, in the spirit of [7], gradient fatigue criteria extended from classical ones with stress gradient terms are proposed and validated to clarify the issues. The main idea is to maintain the general framework of the classical fatigue criteria, but to embed into it gradient terms which enable to describe the effects concerning the stress heterogeneous distribution. Three steps are done: first, the dependence of fatigue limit on the previous factors in the cases of uniaxial stress cyclic loadings is phenomenologically analyzed; second, the stress gradient fatigue criteria which capture the previous factors are established; and finally, a generalization to multiaxial loadings is performed and some applications are provided.

The outline of the work is as follows. Section 2 focuses on reanalyzing existing experiments on gradient, size and loading effects; in Section 3, basing on these analyses as well as notable observations and using as a basis classical fatigue criteria in the spirit of [7], new criteria with stress gradient terms entering not only in the normal stress but as well in the shear stress parts, are proposed in the context of macroscopic elasticity. Such a formulation allows the new criteria to capture the phenomena[2] only by means of gradient terms. These criteria are generalized under multiaxial loadings to be a new class of stress gradient multiaxial fatigue criteria; in Sections 4 and 5, some classical fatigue criteria such as Crossland and Dang Van are extended within such framework; Section 6 is devoted to their numerical implementation; and finally, Sections 7 and 8 are discussions and conclusions.

2. Analyses of gradient fatigue tests: size, gradient and loading effects

In this section, analyses on single component zero and non-zero gradient fatigue tests from the literature, including two groups, uniaxial normal stress and shear stress tests, are made to clarify the size, stress gradient and loading effects on fatigue limits. The tests exempt from the size and gradient effects, are used as reference. A special attention is also paid on the interpretation of the three effects and their relation as well as the capacity of either eliminating or integrating them into "gradient terms" for some cases. Analyses and preliminary conclusions drawn here for single component fatigue tests are generalized to formulate new gradient fatigue criteria under multiaxial cyclic loadings.

2.1. Uniaxial normal stress cyclic loading

2.1.1. Experimental observations and interpretation of stress gradient effect

Some analyses of [7,13] are reported here on fatigue endurance of metals in bending or tension–compression tests. Two respective distinct groups of results, uniaxial normal cyclic stress states with non-zero and zero normal stress gradients, respectively, allow to draw some comments about the normal stress gradient effect and about the possibility of integrating the loading effect into gradient effect. In the first example, a well-established experimental fact is always found: for the same smooth geometry and material, and the same nominal stress σ_{max} (Fig. 1(a)), the specimen in fully reversed tension–compression test sustains lower nominal fatigue stress than in fully reversed bending test. Or similarly but in another observation [41,13,7]: a large number of experiments proved that the fully reversed bending fatigue limit f_{-1} (rotative bending, or plane bending) is always higher than its counterpart σ_{-1} in fully

[2] In this study, these effects are captured in the sense that the gradient effect has to be present as prerequisite – to which the loading effect is naturally attached, whereas and the pure size effect is proved unimportant compared to the others.