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Prediction of fatigue crack growth based on low cycle fatigue properties

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ABSTRACT

Theoretical models of the fatigue crack growth without artificial adjustable parameters were proposed by considering the plastic strain energy and the linear damage accumulation, respectively. The crack was regarded as a sharp notch with a small curvature radius and the process zone was assumed to be the size of cyclic plastic zone. The near crack tip elastic–plastic stress and strain were evaluated in terms of modified Hutchinson, Rice and Rosengren (HRR) formulations. Predicted results from two established models have been soundly compared with open reports for frequently used materials. It is found that experimental results agree well with theoretical solutions.

1. Introduction

Fatigue crack growth behaviors have been studied for various types of engineering structural materials in the past several decades. In order to correlate the damage evolution with the cyclic deformation characteristics of a material with macroscopic cracks, notched specimens are typically introduced to calibrate the crack growth rate da/dN [1]. It is well-known that fatigue cracks initiate and propagate due to the local cyclic plastic yielding of the materials for exampling Model I cracking problems [2]. A crack growth law can then be formulated with the help of the stress and strain field ahead of the crack tip together with a suitable failure criterion.

The damage-tolerant design method assumes that engineering components contain intrinsic imperfections in the form of macro cracks. Generally, the cracks can propagate only when a certain critical length is approached. Such a local nature of the fatigue phenomenon can be described using a sigmoidal curve in the fatigue crack growth rate of $\log(da/dN)$ vs. $\log(\Delta K)$. The sigmoidal curve is soundly bounded at the extremes by the lower bound of the fatigue threshold ΔK_{th} and by the upper bound of the critical stress intensity factor range ΔK_c. In the intermediate range, $\log(da/dN)$ is nearly linearly correlated with $\log \Delta K$, as formulated by Paris and Erdogan [3]. Notably, some life relationships between the fatigue crack growth rate and the low cycle fatigue (LCF) properties of the material have also been established. A number of crack growth rate models previously proposed surrounding the crack tip region are individually based on the fatigue ductility [4], the plastic strain energy [5–10] and the weighted value of the local strain [11]. Unfortunately, most of these theoretical models intrinsically contain adjustable material parameters that requires to be determined experimentally or numerically. Although a few equations with material constants have been established to effectively predict the fatigue cracking growth characteristics, the bore some problem is that material constants are especially difficult to be acquired from the viewpoint of cost.

The progression of fatigue cracking is generally assumed to be an incremental growth due to a critical energy accumulated at the crack-tip. The material ahead of the crack is necessarily modeled as an assemblage of uniaxial material elements, and thus the crack growth can be regarded as the successive failure process of these elements. Additionally, the width of the material element is assumed to be the length of cycle plastic zone along the progressive crack direction. More importantly, the success of such a theory depends on the specification of an appropriate plastic strain energy criterion and the linear damage accumulation along the crack growth direction inside of the cyclic plastic zone. As the stress and the strain are theoretically singular at the crack tip, the plastic strain and the linear damage are somewhat difficult to define precisely. However, the singularity will disappear through introducing the crack tip blunting in this study. Therefore, finite values of the stress and the strain are the focus of the present study. Two theoretical models of the fatigue crack growth based on the plastic strain energy and linear damage accumulation therefore were developed inside the low cycle plastic zone at the crack tip, respectively. Theoretical models can accommodate both the intermittent growth and the continuum growth currently identified for the

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majority of engineering materials. Furthermore, these models can be used to analyze the surface-based cracking life of engineering structures such as the nuclear pressure vessels and pipelines, and high pressure storage tanks.

In the following sections, two analytical models based on the plastic strain energy (PSE) and the linear damage accumulation (LDA), respectively, will be thoroughly established, which physically remove some adjustable materials constants for the effective predictions fatigue crack growth (FCG) rate. Two theoretical model formulated here are expected to be an important complement to a relatively reliable solution of in-service fatigue life for engineering materials.

2. Fatigue crack growth models

An analytical solution of the stress and strain distribution ahead of a crack loaded in antiplane shear (Mode III) during the small-scale yielding has been derived by Schwalbe [12]. Based on the Hutchinson, Rice and Rosengred (HRR) field, a similar analytical solution as for the tensile loading (Mode I) that is the most important solution from an engineering application viewpoint, is available for small scale yielding. Despite the significant plastic deformation in the crack tip zone, the linear elastic fracture mechanics (LFEM) approach is frequently regarded as a well-established theoretical method for the small scale yielding analysis.

Consider a material with strain hardening behavior and a defined stress and strain relationship ahead of the crack tip. The following Fig. 1 illustrates the cyclic stress-strain relationship.

Assuming Masing behaviors are satisfied inside of the cyclic plastic zone at the fatigue crack tip. The increasing branches of hysteresis loops for various loading levels should be coincident, thus the stress–strain hysteresis loops can be approximated. The stress and strain range can thus be written as [13,14]

$$\Delta\sigma(r) = 2K'(\Delta\varepsilon_p/2)^{n'} \tag{1}$$

$$\Delta\varepsilon_p(r) = \frac{2\sigma_{yc}}{E}\left(\frac{r_c}{r}\right)^{1/(1+n')} \tag{2}$$

where $\sigma_{yc} = E\varepsilon_{yc}$ is the cyclic tensile yielding stress, E is the elastic modulus, $\Delta\varepsilon_p$ is the plastic strain range, $\Delta\sigma$ is the stress range, n' is the cyclic strain hardening exponent, K' is the cyclic strain hardening coefficient, r_c is the cyclic plastic zone and r is the distance from the crack tip. The cyclic plastic zone r_c can be formulated by [15]

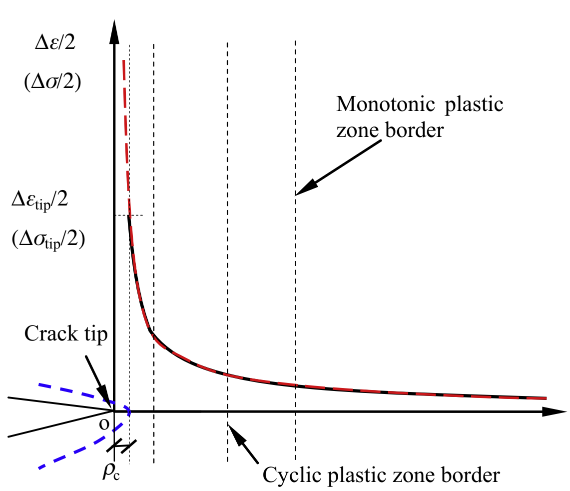

Fig. 1. Distribution of stress and strain at the fatigue crack tip.

$$r_c = \frac{1}{4\pi(1+n')}\left(\frac{\Delta K}{\sigma_{yc}}\right)^2 \tag{3}$$

in which ΔK is the stress intensity range.

2.1. FCG based on a plastic strain energy (FCG-PSE)

In terms of the product of the stress and the strain, Eqs. (1) and (2) can be arranged into the following equation:

$$\Delta\sigma \cdot \Delta\varepsilon_p = 4K'\left(\frac{\sigma_{yc}}{E}\right)^{(n'+1)}\frac{r_c}{r}. \tag{4}$$

The above equation will exhibit a singularity as $r \to 0$. However, such singularity is rather unreasonable due to crack tip blunting from complex material and loading behaviors [16]. With the nature of crack tip blunting, the stress and strain have a finite magnitude in a high strain zone ahead of the crack tip. The cyclic plastic zone is treated as the "process zone" where the majority of the damages occurs. In the process zone, the plastic strain range is much larger than the elastic strain range, thus leading to $\Delta\varepsilon \approx \Delta\varepsilon_p$. Let us now introduce a critical crack blunting radius ρ_c as

$$\rho_c = \frac{1}{4\pi(1+n')}\left(\frac{\Delta K_{\mathrm{th}}}{\sigma_{yc}}\right)^2, \tag{5}$$

where ΔK_{th} is the stress intensity threshold.

The critical radius ρ_c is associated with the threshold stress intensity range. Generally, the fatigue cracks will not propagate when the critical blunting radius is less than the threshold measures from experiments of a material.

Thus Eq. (4) can be rewritten as

$$\Delta\sigma \cdot \Delta\varepsilon_p = 4K'\left(\frac{\sigma_y}{E}\right)^{n'+1}\frac{r_c}{r+\rho_c} \tag{6}$$

From this, it is seen that the product $\Delta\sigma{\cdot}\Delta\varepsilon_p$ in the cyclic plastic zone can be obtained analytically. By integrating within the range $[0,\ r_c - \rho_c]$ using above Eq. (6), the plastic strain energy can be determined by the integral as

$$\int_0^{r_c-\rho_c}\Delta\sigma \cdot \Delta\varepsilon_p \mathrm{d}r = 4K'\left(\frac{\sigma_{yc}}{E}\right)^{(n'+1)} \cdot r_c \cdot (\ln r_c - \ln\rho_c), \tag{7}$$

The fatigue resistance of the material ahead of the crack is governed hypothetically by the local state of stress and strain range perpendicularly to the cracking growth examplied for Mode I problems. A fatigue failure criterion can be applied in the cyclic plastic zone, r_c. Based on the Manson-Coffin strain/life relationship obtained from smooth specimens, strain/life relationship can be described either in terms of the plastic strain or stress range and the number of cycles to failure $2N_f$. Taking the detrimental effect of a mean stress into account since Morrow's work [15], the superimposition yields the following relationship between the total stain amplitude and the number of cycles to technical failure N_f.

$$\Delta\sigma = 2(\sigma'_f - \sigma_m)(2N_f)^b \tag{8}$$

$$\Delta\varepsilon_p = 2\varepsilon'_f(2N_f)^c \tag{9}$$

in which ε'_f and σ'_f are the fatigue ductility and strength coefficients, respectively, exponents b and c are the undetermined material properties and the σ_m is the mean stress. The product of the plastic strain and stress range can be calculated using both Eqs. (8) and (9):

$$\Delta\sigma \cdot \Delta\varepsilon_p = 4(\sigma'_f - \sigma_m)\varepsilon'_f(2N_f)^{b+c}. \tag{10}$$

By rearranging this Eq. (10) and the processing zone, $r_c - \rho_c$, we can obtain the following relationship,

$$\Delta\sigma \cdot \Delta\varepsilon_p \cdot (r_c - \rho_c) = 4(\sigma'_f - \sigma_m)\varepsilon'_f(2N^*)^{b+c} \cdot (r_c - \rho_c), \tag{11}$$