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# Fatigue crack growth in a diverse range of materials

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ABSTRACT

This paper generally examines long crack growth data for a range of aerospace and rail materials tested at a variety of R ratios. The results of this study revealed that, for the 22 materials studied, the crack growth rate, $da/dN$, could be represented by a variant of the Hartman and Schijve equation with $da/dN$ being proportional to the quantity $(\Delta K - \Delta K_{th})^{\alpha}$ where $\alpha$ is approximately two. For cracking in 7050-T7451 it was also shown that this formulation holds for both long and small cracks, although a different value of $\Delta K_{th}$ was required for small cracks compared to that required for long cracks. A possible explanation for this discrepancy is proposed and a methodology to estimate the small crack behaviour based on long crack data is presented. As such these observations have the potential to simplify the prediction of fatigue crack growth lives.


## 1. Introduction

The compendium of F/A-18 Hornet fatigue crack growth data by Molent et al. [1] examined more than 350 different cracks in 7050-T7451, other 7000 series aluminium alloys, Mil-annealed Ti–6Al–4V titanium and AF1410 steel that arose in a variety of full scale fatigue tests and the associated coupon tests. On examining the crack length versus cycle data, it was found that, in these tests, the majority of the fatigue life was generally consumed in the short crack regime, i.e. in growing a crack to a size of approximately 1 mm. It was also found that in almost all cases there was a near linear relationship between the log of the crack length/depth and the number of applied load blocks and that this relationship held from a starting length of less than 100 μm to lengths in excess of 5 mm's. These findings led to the development of an effective block approach (EBA) [2] to predicting crack growth in combat aircraft in service with the Royal Australian Air Force (RAAF). This characteristic has also resulted in a re-examination of some of the current crack growth models [2–5].

The EBA formulation treats a block of spectrum loading as a single cycle and has the following modified 'Paris' form:

$$\frac{da}{dt} = C_{VA}(K_{Ref})^{m_{VA}} \quad (1)$$

where $da/dt$ is the crack growth rate, typically expressed as m/flight hour or m/block, $C_{VA}$ is a spectrum dependent parameter, which has a weak stress dependency [2], $m_{VA}$ is a constant and

$$K_{Ref} = \sigma_{Ref}\beta\sqrt{(\pi a)} \quad (2)$$

where $\sigma_{Ref}$ is an arbitrary reference stress that is taken from the block spectrum (e.g. the peak stress or the rms value of the stress in the spectrum), and $\beta$ is the geometry and loading factor. In [2,3] the exponent $m_{VA}$ in Eq. (1) was found to be approximately 2.

In this context the present paper examines a variant of the Hartman and Schijve crack growth equation [6] which states that for constant amplitude loading the increment in the crack length per cycle, $da/dN$, is a linear function of $(\Delta K - \Delta K_{th})^{\alpha}$. Here $\Delta K = (K_{max} - K_{min})$ is the range of the stress intensity factor in the cycle, $K_{max}$ and $K_{min}$ are the maximum and minimum values of the stress intensity factor in the cycle, $\Delta K_{th}$ is the fatigue threshold and $\alpha$ is a material constant. As a result of this study we find that in 22 arbitrarily chosen different metallic materials, viz: the aluminium alloys 2024-T3, 2024-T351, 2025-T6, 2219-T851, 7075-T6, 7050-T7451 and 7050-T76511, in D6ac, 4340 and 10Ni–8Co–1Mo steels, in Mil-annealed Ti–6Al–4V, STOA Ti–6Al–4V, a Ti–6Al–4V titanium forging, Ti-62222, in the rail steel L6B2, in Class MC and Class B rail wheel steels, a range of Sumitomo Metal Technology experimental low carbon martensitic/bainitic rail wheel steels, B+ steel (a steel used in rail rolling stock) and the rail car tank steel TC-128B, tested at a variety of R ratios in ambient air the crack growth could be represented by a variant of the Hartman and Schijve equation such that $da/dN$, is a linear function of

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$(\Delta K - \Delta K_{th})^{\alpha}$ with $\alpha \approx 2$, as suggested by Liu and Liu [7]. For these data it may be expected that adopting fracture mechanics using the stress intensity factor is a relevant approach.

We also see that for cracking in 7050-T7451 this formulation holds for both long and small cracks. As such these observations may have the potential to simplify the prediction of fatigue crack growth lives. In this context Wanhill [8] has noted that for small cracks that initiate from small naturally occurring initial discontinuities the fatigue threshold $\Delta K_{th}$ is very small. In such cases (with the exceptions of the initial crack growth period when the flaw is still influenced by the nature of the initiating defect, and the region when $K_{max}$ is approaching its cyclic fracture toughness ($K_{cy}$)) when $\alpha \approx 2$ the linear relationship between $da/dN$ and $(\Delta K - \Delta K_{th})^{\alpha}$ suggests that the crack length versus cycles history will be (near) exponential. In this context it should be noted that approximately exponential crack growth has been previously noted for a wide range of materials tested under both constant and variable amplitude loading [1–5,9–14].

## 2. The Hartman–Schijve equation

It is commonly thought that the increment in the crack length per cycle, $da/dN$, can be related to $\Delta K$ and/or the maximum stress intensity factor $K_{max}$. This approach was first suggested in 1961 by Paris et al. [15], who related ($da/dN$) to $K_{max}$. Paris and Erdogan [16] subsequently noted that the work of Liu [17] implied that crack growth was a function of $\Delta K$. They then proposed [18] the well known Paris equation:

$$da/dN = C(\Delta K)^{m} \tag{3}$$

where $C$ and $m$ are experimentally obtained, and are considered to be a constant for a particular material and environment. The difference between the work of Liu [17] and that of Paris and Erdogan [18] was that, as explained in [16], in Liu's work m was equal to 2, whilst Paris and Erdogan analysed measured data from three different configurations and sources and determined a value for $m$ of 4 for 7075-T6.

Forman et al. [19] extended the Paris equation to account for $K_{max}$ approaching its fracture toughness $K_c$, viz:

$$\frac{da}{dN} = \frac{C(\Delta K)^{m}}{((1-R)K_C - \Delta K)} \tag{4}$$

where $R = K_{min}/K_{max}$. Hartman and Schijve [6] subsequently suggested that $da/dN$ should be dependent on the amount by which $\Delta K$ exceeds the fatigue threshold $\Delta K_{th}$ of the material under the associated test environment, i.e. $\Delta K - \Delta K_{th}$. This led them to develop a modification to the Forman equation, viz:

$$\frac{da}{dN} = \frac{D'(\Delta K - \Delta K_{th})^{\alpha}}{((1-R)K_C - \Delta K)} \tag{5}$$

where $D'$ is a constant for a given value of $R$, and $\alpha$ is a constant.

In this paper we build on the Hartman–Schijve concept, i.e. that $da/dN$ should be dependent on the amount by which $\Delta K$ exceeds the fatigue threshold $\Delta K_{th}$ of the material under the associated test environment, to investigate a variant of the original Hartman–Schijve equation, viz:

$$da/dN = D(\Delta\kappa)^{\alpha} \tag{6}$$

where the crack driving force, $\Delta\kappa$, is taken to be

$$\Delta\kappa = (\Delta K - \Delta K_{thr})/\sqrt{(1 - k_{max}/A)} \tag{7}$$

where $\Delta K_{thr}$ is the fatigue threshold and the subscript $r$ has been used to indicate that it is a function of both the $R$ ratio and the crack length.

This equation has the advantage that it allows the constant $\alpha$ to be determined directly from the slope of the $da/dN$ versus $\Delta\kappa$ curve. Furthermore for the special case when $\alpha = 2$ Eq. (6) takes on a similar form to that of Eq. (5). Here the constant A may be considered to represent an apparent $K_{cy}$ as defined in [20,21] which may differ from the fracture toughness ($K_c$) obtained from static testing, see [21]. (If the units of $da/dN$ are m/cycle and $K$ is in MPa $\sqrt{m}$ then the units of $D$ are $MPa^{-\alpha}\,m^{1-\alpha}\,cycles^{-1}$.) The role of the denominator in Eqs. (5) and (6) is to match the predicted and measured crack growth rates for large values of $K_{max}$.

Liu and Liu [7] revealed that da/dN should always be proportional to $(\Delta K - \Delta K_{th})^2$. Their equation took the form:

$$\frac{da}{dN} = [0.036/(E\sigma_y)](\Delta K - \Delta K_{th})^2 \tag{8}$$

where $E$ is the Young's modulus and $\sigma_y$ is the yield stress.

For the one particular problem used in [6] to evaluate Eq. (5) the exponent $\alpha$ was also found to be approximately 2. An exponent $\alpha$ of approximately 2 was also found for short crack growth in a weldable-structural C-Mn steel by Smith and Smith [22]. In such cases, i.e. when $\alpha = 2$, Eqs. (5) and (6) both have $da/dN$ being proportional to $(\Delta K - \Delta K_{th})^2$. Furthermore, when $\alpha = 2$ and the value of $K_{max}$ is small then the term $K_{max}/A$ in Eq. (7) may be neglected. This suggests that when $\alpha = 2$ that

$$D = [0.036/(E\sigma_y)] \tag{9}$$

may represent a reasonable first approximation. This approximation will be investigated later in the paper.

The linear dependence of $da/dN$ on $(\Delta K - \Delta K_{th})^2$ was also reported in the paper by Miller and Gallagher [23] who studied cracking in 2219-T851. Indeed, a linear dependency of $da/dN$ upon $\Delta K^2$, for Region II crack growth where the term $\Delta K_{th}$ plays only a minor role, was also reported in a USAF study [24]. Similar dependencies on $(\Delta K - \Delta K_{th})^2$ were also reported in [25,26] where it was shown that $da/dN$ could often be expressed as a function of $(\Delta K_{eff} - \Delta K_{eff,th})^2$ where $K_{eff}$ was an effective stress intensity factor and $\Delta K_{eff}$ was the associated threshold.

Whilst it is well known that $\Delta K_{th}$ is a function of the $R$ ratio, Frost [27] was the first to reveal that $\Delta K_{th}$ is also a function of the crack length and that $\Delta K_{th}$ decreases as the crack length decreases. This phenomenon was subsequently confirmed by Usami and Shida [28] and Kitagawa and Takahashi [29]. Indeed, Kitagawa and Takahashi [29] established that there is a transition crack length, below which $\Delta K_{th}$ is smaller than its value for long cracks. As such Eqs. (5), (6), and (8) imply that, in general, $da/dN$ is a function of $\Delta K$, the $R$ ratio and the crack length. This crack length dependency was clearly stated in [25,26].

### 2.1. The fatigue threshold $\Delta K_{th}$

Before examining the ability of Eq. (6) to represent crack growth we need to discuss the dependency of the fatigue threshold $\Delta K_{th}$ on specimen geometry as well as on the test method. Although in the development of the ASTM test standards [30] it was found that the value of $\Delta K_{th}$ was specimen geometry independent subsequent tests [4,31–35] have shown that this is not always the case. It has also been suggested [36–38] that the ASTM load reducing test methodology can produce erroneous values for $\Delta K_{th}$. However, this conclusion was questioned in [39]. Consequently, in this study we will limit the data sets studied to tests that are either constant amplitude, $K_{max}$, or load increasing tests. However, the data presented in [32] reveals that for high $R$ ratio tests the measured value of $\Delta K_{th}$ appears to be less geometry and test methodology dependent. Consequently in those few cases where constant amplitude, $K_{max}$, or load increasing test data was not available we analysed high $R$ ratio load reducing tests, i.e. $R = 0.5$ or greater. Similarly