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A novel, single-layer model for composite plates using local-global approach

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ABSTRACT

In structural analysis, many components are approximated as plates. More often that not, classical plate theories, such as Kirchhoff or Reissner-Mindlin plate theories, form the basis of the analytical developments. The advantage of these approaches is that they leads to simple kinematic descriptions of the problem: the plate's normal material line is assumed to remain straight and its displacement field is fully defined by three displacement and two rotation components. While such approach is capable of capturing the kinetic energy of the system accurately, it cannot represent the strain energy adequately. For instance, it is well known from three-dimensional elasticity theory that the normal material line will warp under load for laminated composite plates, leading to three-dimensional deformations that generate complex stress states. To overcome this problem, several layer-wise plate theories have been proposed. While these approaches work well for some cases, they often lead to inefficient formulations because they introduce numerous additional variables. This paper presents a novel, single-layer theory using local-global Approach: based on a finite element semi-discretization of the normal material line, the two-dimensional plate equations are derived from three-dimensional elasticity using a rigorous dimensional reduction procedure. Three-dimensional stresses through the plate's thickness can be recovered accurately from the plate's stress resultants.

1. Introduction

Plates are structural components for which one dimension is far smaller than the other two. The mid-plane of the plate lies along its two long dimensions, and the normal to the plate extends along the shorter dimension. The material properties of the plate are assumed to vary smoothly over the plate's mid-plane surface.

Numerous structures can be approximated as plates or shells. The long, slender wings of an aircraft can be analyzed, to a first approximation, as beams, but a more refined analysis will treat the upper and lower skins of the wing as thin plates or shells supported by ribs and longerons or stiffeners. The same can be said about helicopter or wind turbine blades. Buckling of the face sheets of wind turbine rotor blades is an important problem that cannot be captured by beam models. This instability, however, will be captured by plate models.

Solid mechanics theories describing plates, more commonly referred to as "plate theories," play an important role in structural analysis because they provide tools for the analysis of these commonly used structural components. Although more sophisticated formulations, such as three-dimensional elasticity theory, could be used for the analysis of plates and shells, the associated computational burden is often too heavy. Plate theories reduce the analysis of complex, three-dimensional structures to two-dimensional problems. The main goal of the proposed plate theory is to approximate the three-dimensional plate-like structure with a two-dimensional model, while retaining an accurate representation of the local, three-dimensional stress and strain fields through the thickness of the plate.

Several plate theories have been developed based on various assumptions, and lead to different levels of accuracy. One of the simplest and most useful of these theories is due to Kirchhoff who analyzed the bending of thin plates. Kirchhoff plate theory (Timoshenko and Woinowsky-Krieger, 1959; Bauchau and Craig, 2009) is used commonly in many civil, mechanical and aerospace applications, although shear deformable plate theories (Reissner, 1945, 1947; Mindlin, 1951), often called "Reissner-Mindlin plates,"

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have also found wide acceptance.

In many applications, however, plates are complex built-up structures. In aeronautical constructions, for instance, the increased use of laminated composite materials leads to heterogeneous, highly anisotropic structures. Layers of anisotropic material are stacked through the thickness of the plate. This new type of structural component prompted the development of new plate theories (Librescu, 1975; Whitney, 1987; Reddy, 1996), often based on classical lamination theory (Christensen, 1979; Tsai and Hahn, 1980).

Further refinements then followed with the goal of capturing the intricate three-dimensional stress field that develops under load, with special emphasis on interlaminar shear stresses. The various approaches fall into two categories: single-layer and layer-wise plate theories. Single-layer plate theories have been proposed by assuming higher-order or zigzag displacement fields for the entire normal material line (Lo et al., 1977; Reddy, 1984; Pandya and Kant, 1988; Murakami, 1986; Cho and Parmerter, 1993). In layer-wise theories (Sciuva, 1992; Carrera, 1998; Cho and Averill, 2000), the displacement field of the normal material line for each layer is independent of that of the others, with a simple C_0 continuity conditions imposed at the layer boundaries. Although these approaches lead to higher accuracy, the number of unknowns increases considerably. More recently, Carrera (Carrera, 2001; Carrera and Demasi, 2002) proposed the "Carrera Unified Formulation" for the general description of two-dimensional formulations for multilayered plates and shells. This formulation is composed of a series of hierarchical models from simple, single-layer models up to higher-order layer-wise descriptions and provide a tool for the systematic assessment of different theories.

For plates, the aspect ratio is usually a small parameter and hence, the stress and displacement gradients over the plate's mid-surface are often smaller than those through its thickness. Based on this assumption, efficient single-layer plate models can be derived rigorously from three-dimensional elasticity through dimensional reduction techniques that split the original problem into a two-dimensional analysis over the plate's mid-plane surface and a one-dimensional, through-the-thickness linear analysis. The plate's stiffness matrix is a by-product of the dimensional reduction process, which also enables the recovery of three-dimensional stress fields. These approaches, derived from three-dimensional elasticity theory directly, can handle laminated plates and shells made of anisotropic composite materials without increasing the total number of unknowns.

Asymptotic and multiscale analysis methods have been the tools of choice for the derivation of single-layer models. Asymptotic and multiscale methods expand the solution in terms of the aspect ratio, leading to a rational decomposition of the three-dimensional problem into two-dimensional equations over the plate's mid-plane and a through-the-thickness problem. Based on this approach, Friedrichs and Dressler (Friedrichs and Dressler, 1961) and Kalamkarov (Kalamkarov, 1992) investigated isotropic and inhomogenous plate problems, respectively. A unified theory based on Variational Asymptotic Method (Berdichevsky, 1979, 1982), (VAM), presenting both linear, one-dimensional through-the-thickness analysis, and nonlinear, two-dimensional analysis over the mid-plane surface of the plate or shell was developed by Sutyrin and Hodges (Sutyrin and Hodges, 1996; Sutyrin, 1997), and Yu et al. (Yu et al., 2002; Yu and Hodges, 2004). More recently, Kim (Kim, 2010) proposed a finite element based asymptotic analysis for generally anisotropic plates.

It is not necessary to use asymptotic expansion methods to tackle plate analysis. A finite element based, semi-discretization approach was proposed Masarati and Ghiringhelli (Masarati and Ghiringhelli, 2005) to solve laminated plate problems; they found solutions of the three-dimensional equilibrium equations through the thickness of the plate. These solutions then yield the plate's global compliance matrix and the local stress field can be recovered.

The Representative Volume Element (RVE) approach is a common tool for multiscale analysis. Gruttmann and Wagner (Gruttmann and Wagner, 2013) used a through-the-thickness RVE to develop local/global plate models. The displacement field of the RVE is split into rigid normal material line and warping components. The local RVE and global plate models are coupled at the lateral surfaces of the RVE and are solved simultaneously.

In this paper, a novel single-layer model for composite plates is developed by assuming the gradients of stress and displacement components over the plate's mid-surface to be smaller than those through its thickness. A semi-discretization of the general equations of three-dimensional elasticity is performed, defining the "local model." The equations relating the stress resultants, the plate's deformation measures, and the warping field of the normal material line are derived from a linear combination of the equations of the local model. These equations define the relationship between the stress resultants and deformation measures in an implicit manner, and hence, define the "global model." By means of a recursive process, power series solutions are found for the combined equations of the local and global models. Based on these solutions, the local problem for plate-like structures is reduced to the corresponding global problem, the single-layer model, and the local stress and strain fields can recovered from the global solution. The 8×8 stiffness matrix of the plate is a by-product of the reduction process; it takes into account warping effects due to material heterogeneity. Local stress and strain fields at any point through the thickness of the plate can be recovered from the global solution. The proposed method is applicable to anisotropic plates with arbitrarily complex through-the-thickness lay-up configurations.

2. Kinematics of the problem

Fig. 1 depicts a flat plate of thickness h and a typical material line, denoted $\mathcal{L}$, normal to the midplane of the plate, $\mathcal{S}$. The volume of the plate, denoted as Ω, is generated by sliding the normal material line over the plate's midplane, *i.e.*, $\Omega = \mathcal{S} \times \mathcal{L}$. The plate's lateral boundary, $\partial\Omega$, is generated by sliding the normal material line over the midplane's boundary Γ, *i.e.*, $\partial\Omega = \Gamma \times \mathcal{L}$. Coordinates η_1 and η_2 define the plate's midplane, *i.e.*, they measure length along unit vectors $\bar{b}_1$ and $\bar{b}_2$, respectively. Point **B** is located at the intersection of the normal material line with the plate's midplane. The unit tangent vectors to midplane are defined as $\bar{b}_1 = \partial \underline{r}_B / \partial \eta_1$

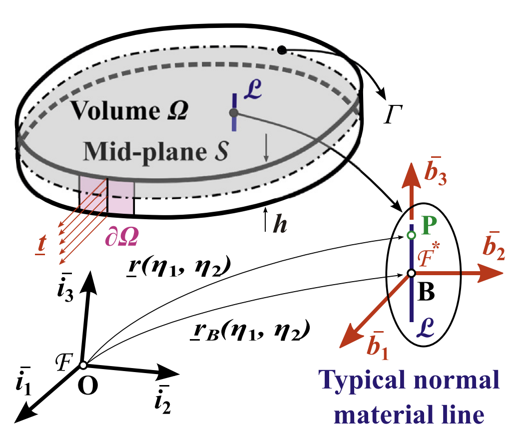

Fig. 1. Configuration of the plate.