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Problem of axisymmetric plane strain of generalized thermoelastic materials with variable thermal properties

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ABSTRACT

This paper is concerned with asymptotic solutions for the axisymmetric plane strain problem with variable material properties in the context of different generalized models of thermoelasticity. The unified forms of the governing equations for axisymmetric plane strain problem, involving the generalized theory with one thermal relaxation time (L-S theory), the generalized theory with two thermal relaxation time (G-L theory) and the generalized theory without energy dissipation (G-N theory), are presented by introducing the unifier parameters. The Laplace transform techniques and the Kirchhoff's transformation are used to obtain the general solutions for any set of boundary conditions in the physical domain. The asymptotic solutions for a specific problem of an infinite cylinder, formed of an isotropic homogeneous material with variable thermal material properties, whose boundary is subjected to a sudden temperature rise, are derived by means of the limit theorem of Laplace transform. In the context of these asymptotic solutions, some generalized thermoelastic phenomena are obtained and illustrated, especially the jumps at the wavefronts, induced by the propagation of heat signal with a finite speed, are also observed clearly. By the comparison with the results obtained from the case of constant material properties, the effect of variable thermal material properties on the thermoelastic behavior is also discussed.

1. Introduction

Due to the widespread use of some new processing techniques such as pulse laser irradiation and the rapid solidification, which can supply heat pulses in a very fast time (Qiu and Tien, 1993), the prediction of thermoelastic behavior involving rapid transient heat conduction is of considerable practical importance in some engineering sciences, such as aerospace, nuclear reactor, etc. Some experiments (Mitra et al., 1995) have proved that the heat signal propagates in an elastic medium with a finite speed once the heat conduction occurs at a low temperature or rapid thermal duration. The conventional coupled theory, proposed by Biot (1956), with the assumption that the thermal propagation velocity is infinite, cannot be an accurate description of this thermoelastic behavior. Therefore, some research effort form a different perspective, using a wave-type equation, and a finite propagation velocity of the heat wave, were proposed to overcome the shortcomings inherent in the conventional coupled theory by different researchers (Lord and Shulman, 1967; Green and Lindsay, 1972; Green and Naghdi, 1993; Povstenko, 2011; Wang and Song, 2012). In accordance with these modified models, named as generalized theories to distinguish from the conventional theory, the second sound effect (Chandrasekhariah, 1986), as well as other generalized thermoelastic phenomena, induced by the propagation of heat signal with a finite speed, have been studied to some extent (Chandrasekhariah, 1998; Hetnarski and Ignaczak, 1999; Tian and Shen, 2012).

It worth noting that these generalized theories, used to investigate the rapid transient heat conduction, have not involved the material properties, which limited the applicability of these results obtained from these generalized models to certain ranges of temperature (Ezzat et al., 2004). The mechanical and thermal properties are usually changed with temperature for most materials, and this temperature-dependent property is more significant at high temperatures. Thus, it has become necessary for the research of

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thermoelastic behavior to take into account the real behavior of material properties. Since the experimental results (Wang, 1987) show these variable material properties, such as the modulus of elasticity, the conductivity and the specific heat, are usually changed with the temperature, some thermoelastic problems involving variable material properties have been investigated in the context of different generalized theories (Ezzat et al., 2004; Youssef, 2005; Aoudai, 2006; Allam et al., 2010; Akbarzadeh et al., 2011; Sherief and Abd El-Latief, 2013), where an assumption that the material properties are the linear function of the reference temperature was used to simplify the solution of governing equations. Xiong and Tian (2011) considered the real relation between the material properties and actual temperature to solve a magneto-thermoelastic problem of a semi-infinite body with voids, and the similar results for the effects of the temperature dependency on thermoelastic behavior with above investigations were obtained.

Since the governing equations of these generalized models are complicated for a general case, especially for these involving variable material properties, the ability to solve these equations efficiently is very important in revealing the generalized thermoelastic phenomena without the support of test results. The integral transform technique and its numerical inversion is the common method used in preceding works (Ezzat et al., 2004; Youssef, 2005; Aoudai, 2006; Allam et al., 2010; Akbarzadeh et al., 2011; Sherief and Abd El-Latief, 2013). The truncation error and the discretization error, however, generated from the numerical inversion, would reduce the precision of final calculations and therefore cannot effectively predict the wavelike behavior of heat conduction. The other method used to solve these governing equations is the direct numerical solutions in the time domain by means of different numerical techniques, such as the finite element method (Tian et al., 2006; Xiong and Tian, 2011). The advantage for this method is to avoid the complicated integral transform and corresponding inverse transform, and some specific problems with complicated boundary conditions can also be solved. However, the shortcomings inherent in the numerical techniques, such as the dependency of the difference schemes, the reliability of the meshing of grids and the calculation errors, would weaken the application of these generalized theories (El-Karamany and Ezzat, 2004). Recently, an asymptotic method, based on the limit theorem of Laplace transformation, has been introduced to solve some thermoelastic problems in the context of different generalized theories (Balla, 1991; Wang and Song, 2012). By the application to some transient thermal shock problems (Wang and Song, 2012, 2013a) and the problem with variable material properties (Wang et al., 2013b), some generalized thermoelastic phenomena, that cannot be accurately predicted by the preceding two methods, are observed clearly. Meanwhile, the explicit expressions for the propagation of heat signal are also obtained, which is very important to reveal the effect of each characteristic factor involved in these generalized theories.

In this paper, an axisymmetric plane strain problem with variable thermal material properties is investigated in the context of different generalized theories. The unified forms of governing equations, involving the L-S, G-L and G-N models, are derived by introducing the unifier parameters. An asymptotic method is introduced to obtain the unified solutions for these generalized models. The specific problem of an infinite cylinder when the boundary is subjected to a sudden temperature rise is solved by these asymptotic methods, where the variable thermal material properties, including the thermal conductivity and the specific heat, are considered. The explicit expressions for the propagation velocity of the thermal wave and thermoelastic wave, as well as the locations of each wavefront are obtained. The distributions of the displacement, temperature and each stress component are plotted and discussed. The comparison with the results obtained from the case of constant material properties is also conducted to evaluate the effects of variable material properties in the context of different generalized theories.

2. Unified formulation of generalized thermoelasticity

Due to the L-S, G-L and G-N theories of generalized thermoelasticity (Lord and Shulman, 1967; Green and Lindsay, 1972; Green and Naghdi, 1993), the fundamental equations for the isotropic homogeneous material with a unified form are present by introducing the terms η_1 and η_2 as unifier parameters. These equations in general form can be expressed as:

Equation of motion without body force

$$\rho \ddot{u}_i = \sigma_{ij,j}. \tag{1}$$

Constitutive equations

$$\sigma_{ij} = \lambda \gamma_{kk} \delta_{ij} + 2\mu \gamma_{ij} - \beta \left(\theta + \tau_1 \dot{\theta} \right) \delta_{ij}. \tag{2}$$

Linear strain-displacement relations

$$\gamma_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right). \tag{3}$$

Energy equation

$$q_{i,i} = \rho r - \rho c_p \left(\dot{\theta} + \tau_2 \ddot{\theta} \right) - T_0 \beta \dot{\gamma}_{kk} + c_i \dot{\theta}_{,i}. \tag{4}$$

Heat conduction equation

$$\eta_1 q_i + \tau_0 \dot{q}_i + \eta_2 \dot{q}_i = -\eta_1 k \theta_{,i} - \eta_2 k^* \theta_{,i} - c_i \dot{\theta}. \tag{5}$$

In the preceding equations, u_i are the components of the displacement vector, q_i are the components of heat flux vector, σ_{ij} are the components of the stress tensor, γ_{ij} are the components of the strain tensor, $\theta = T - T_0$ is the increment temperature, T is the absolute temperature, T_0 is the reference temperature, ρ is the mass density, k is the thermal conductivity, c_p is the specific heat at constant strain, r is the internal heat source, $\beta = (3\lambda + 2\mu)\alpha_T$ is the thermal-mechanical coefficient, α_T is the coefficient of linear thermal expansion, λ and μ are the Lame constants, τ_0, τ_1 and $\tau_2$are the relaxation time constant for the L-S and G-L models, respectively, c_i are the components of new material constant proposed in the G-L model, and for the isotropic material $c_i = 0$, and k^* is the new thermal conductivity associated with the G-N model. Meanwhile, the superscript dot $(\cdot)$ and the subscript comma (,) denote the derivatives to the time t and coordinates $x_i (i = 1,2,3)$, respectively.

Eqs. (1)–(5) can be reduced to the governing equation of the L-S, G-L and G-N theories separately by choosing different values of unifier parameters and corresponding material constants.

1) L-S model: $\eta_1 = 1$, $\eta_2 = 0$, $c_i = 0, \tau_1 = \tau_2 = 0$.
2) G-L model: $\eta_1 = 1$, $\eta_2 = 0, \tau_0 = 0$.
3) G-N model: $\eta_1 = 0$, $\eta_2 = 1$, $c_i = 0, \tau_0 = \tau_1 = \tau_2 = 0$.

In addition, if $\eta_1 = 1$, $\eta_2 = 0$, $c_i = 0$ and $\tau_0 = \tau_1 = \tau_2 = 0$, Eqs. (1)–(5) can be reduced to the governing equations of the conventional coupled theory of thermoelasticity (Biot, 1956).

3. Governing equations for axisymmetric plane strain problem with variable thermal material properties

We consider an infinite cylinder formed of an isotropic homogeneous material with variable thermal conductivity and specific