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The method of a floating frame of reference for non-smooth contact dynamics

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A B S T R A C T

A method of a floating frame of reference that performs splitting of a deformable solid into rigid and deforming parts is presented within the context of non-smooth contact dynamics. The decomposition is made in such a way that the deforming part of the velocity field does not contribute either to the motion of the center of mass or the rotational motion. The corresponding numerical method that computes both rigid and deforming motions is presented and extended to multi-body dynamics simulation allowing non-smooth contact interactions, such as impacts and friction. Numerical experiments, where the method is compared with a more traditionally used Total Lagrangian method, justify its preference as a more efficient tool for the simulation of assemblies of stiff and massive objects.

1. Introduction

This paper presents and studies the application of the method of a floating frame of reference to a simulation of solids, participating in non-smooth dynamics.

The method of a floating frame of reference (for simplicity, it shall be referred to as FFR) is a special type of a general family of so called corotational methods. The key idea behind a corotational method is a kinematical splitting into two of the reference configuration of an element of a structure primarily discretized with the Finite Element method (FEM). These are the base configuration and the corotated, or dynamic one. The base configuration is kept fixed for the entire structural analysis, while the corotated configuration is a result of the rigid body motion, i.e. superposition of translation and rotation, of the base configuration. In general, the dynamic configuration is *element dependent* and is defined for each element separately. A far from complete list of the works on the topic includes (Crisfield, 1990; Bergan and Horrigmoe, 1976; Rankin and Brognan, 1986; Rankin and Nour-Omid, 1988; Belytschko and Hsieh, 1973; Simo, 1985; Devloo et al., 2000; Areias et al., 2011; Alsafadie et al., 2010; Felippa and Haugen, 2005).

The FFR method differs significantly from general corotational methods by the fact that it requires only one dynamic configuration for each element. Even more, this single moving frame of reference is introduced without any connection to the elements or any other type of structural discretization of the described solid, and therefore can be defined before even considering discretization in general. The FFR method is intuitively more attractive and has been known for over a century. It has been mostly used for computations in flexible multi-body dynamics, where separate solids are connected via bilateral constraints, typically smooth, (Cardona and Geradin, 2001), (Veubeke, 1976). A brief overview of the method in this area was given in (Shabana et al., 2007).

Variants of the FFR method are distinguished by the type of attachment between the moving frame and the deformable body itself. This is governed by the reference conditions (Schwertassek et al., 1999). Depending on the selected reference conditions, the translational and rotational coordinates associated with the body frame vary differently during the motion. The motion of the selected frame is referred to as reference motion. For example, reference conditions result in the tangent frame, the chord frame or the free frame, among others (Escalona et al., 2003). Reference conditions are related to the boundary conditions that the selected deformation shape functions must fulfil.

Our area of interest is a simulation of assemblies of bodies undergoing non-smooth contact interactions such as shocks and friction, with typically natural external forces such as gravity. A well known method to simulate dynamics of large assemblies of objects

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with such interactions is the Discrete Element method (DEM) (Cundall and Hart, 1992). In DEM, typically only a minimal resolution of the internal deformations are performed as to allow more computational resources for the overall system dynamics. The areas of application of DEM include rocking avalanches (Banton et al., 2009), (Manzella and Labiouse, 2009), masonry structures, granular systems (Ghaboussi and Barbosa, 1990). Even the most sophisticated continuous flow models usually fail to replace DEM in representing accurate physical phenomena. As opposed to DEM, in which the number of bodies ranges from thousands to hundreds of thousands or even more (Mishra and Rajamani, 1992), we are assuming a significantly smaller number of the participating solids, though with the growing computer performance this number may naturally grow as well. The bodies are assumed to fall under an important restriction to eliminate a possibility of nonlinear deformational behavior, such as bending. Hereafter bodies are seen as massive blocks, which means no extremely thin bodies are considered. This restriction also imposes large stiffness on the participating bodies, for which the Young modulus is typically positioned around values of order 10^9–10^{10} or larger. Unlike DEM, we consider more internal degrees of freedom for the solids, which explains our limitation in the number of the solids in the multibody dynamics but allows a more accurate simulation of these solids.

This article focuses on the implementation of the FFR method in the dynamics of several bodies and its integration into the non-smooth contact framework pioneered by J. J. Moreau (1988), (2004) and M. Jean (1999). This is a vastly growing field, and the number of methods has been developed for that framework. Unfortunately, the absolute rigidity model for interacting bodies in a studied collection as a simplification of the large stiffness model may create indeterminacy (plurality of solutions) (Alart, 2014), partly because of the specific nature of the employed interaction laws. One of the ways to treat this problem is by incorporating a finite yet large stiffness for the bodies, and therefore applying the FEM analysis. This bears the most general solver relying on the Total or Updated Lagrangian approach, also known in the engineering community as the method of a large transformation and employing the absolute nodal coordinate formulation (ANCF). The method without any regard to contact is thoroughly described in (Belytschko et al., 2000) and its implementation into contact problems may be found in (Acary and Brogliato, 2008), (Koziara and Bicanic, 2008). With an assumption of a small rotation, the nonlinearity of the method is neglected, and it is renamed into the method of small deformation.

Within the FFR framework, we shall be using the so-called deformable body mean axis frame (Agrawal, 1984). When using this frame, the degree of coupling between the reference coordinates and the elastic coordinates is minimum, though not zero. Neither the corotational method, nor the large transformation approach seem as attractive for the non-smooth dynamics of stiff massive solids as the FFR method with this frame. Assume the solid's position field satisfies $\mathbf{x} = \mathbf{x}(\mathbf{X})$, where $\mathbf{X}$ and $\mathbf{x}$ reside in the base and current configuration respectively. Then the transformation gradient $\mathbb{F}$ may be polarly decomposed as

$$\mathbb{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \mathbb{U}\mathbb{O},$$

where $\mathbb{U}$ is responsible for the deformation, and $\mathbb{O}$ is an orientation as of a rigid body. The assumption for our case implies $\mathbb{O} \approx const$ for the entire structure, therefore the general corotational method, operating with arbitrarily changing $\mathbb{O}$ from element to element, is overused. On the other hand, the large transformation approach is not efficient computationally, due to the presence of the high nonlinearity in the equations even in the absence of contact interactions. The floating frame is intended for tracking the rotational part of the solid, and the nonlinear deformations in this case are completely eliminated from the point of view of that frame, which allows for constant stiffness matrix in the computations throughout the whole simulation. The remaining nonlinearity is only due to rotations, which relaxes an iterative solution process.

Another attractive property of the FFR method is that the floating frame, described above, may provide such characteristics of the moving solid as its center of mass, orientation and angular velocity (without operating with rigid body modes, coming from the FEM discretization), which are the parameters of a purely rigid body and are desirable for intuitive description of deformable, but very stiff solids. For this reason, from now on we shall be referring to the motion of such frame as "rigid motion" of the deformable solid as opposed to more general term "reference motion".

The paper has 4 sections following. As a theoretical base, Section 2 provides a formal kinematical theory for the most general body structure. It is shown that the decomposition into the rigid and the deforming motions for a single solid is always possible in the kinematical sense regardless of the dynamical reasoning for the motion. Although theoretically this splitting is possible for any kind of deforming solids, computationally it is only meaningful in the application area mentioned above, as too much deformation would create nonlinearity and the FFR method would not retain its advantageous status over the method of a large transformation. Note this section is not operating with any sorts of spatial discretizations of a solid and is based purely on the fundamental laws of mechanics. This section may present interest especially for theoretical mechanicians. Section 3 introduces FEM method in the local frame to resolve the deformational behavior and derives a stable second-order accurate Newmark time-stepping scheme and then adapts these results to the non-smooth contact dynamics of a general multi-body system. Finally, Section 4 contains numerical experiments that test the performance and accuracy of the method compared to the large transformation method. Section 5 provides the conclusion and further prospects.

2. The formalism

Everywhere below any bold symbol, for example **a**, describes a vector with at least two components or a set of vectors. Blackboard bold symbols, except for the real number set $\mathbb{R}$, denote operators, tensors (wider than vectors) and their corresponding matrices. Symbol $\times$ denotes vector product and $\otimes$ denotes tensor product of two vectors, i.e. $\mathbf{a} \otimes \mathbf{b} = \mathbf{a}\mathbf{b}^T$. Operation $\mathbb{A} : \mathbb{B}$ returns the sum of all products of the respective elements of the both matrices, i.e. $\sum_{i,j} A_{ij}B_{ij}$.

2.1. Kinematics

Let the Euclidean system of coordinates $Oxyz$ be an inertial frame of reference (Landau and Lifshitz, 1976) called a global frame. For a solid with density ρ and mass m, let $\mathbf{x}(\mathbf{X},t)$ denote a position vector in the global frame of a material point with Lagrange coordinates $\mathbf{X}$ at time t. $\mathbf{v}(\mathbf{X},t)$ denotes the velocity vector of that point. The set of all $\mathbf{X}$ is denoted V and they refer to the initial undeformed (rigid) configuration of the solid.

The main idea behind a floating frame is that any motion of a deformable body V may be described as the superposition of the motions of the *imaginary* rigid and deforming parts (Shabana and Schwertassek, 1998). For a currently used floating orthonormal frame $CXYZ$ and an imaginary rigid body frozen in it, with its center of mass placed at the origin C, the velocity $\mathbf{v}_R$ of that rigid body is