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Local buckling of profiled skin sheets resting on tensionless elastic foundations under in-plane shear loading

Jianghui Dong [a], Yan Zhuge [b], Julie E. Mills [a], Xing Ma [a], *

[a] School of Natural and Built Environments, University of South Australia, Adelaide, SA 5095, Australia
[b] Faculty of Engineering and Surveying, University of Southern Queensland, Toowoomba, QLD 4300, Australia

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ABSTRACT

In this paper, the local buckling behaviour of profiled skin sheets resting on tensionless elastic Winkler foundations under in-plane shear loadings is studied. The profiled sheets are modelled as thin orthotropic plates with the buckling behaviour being expressed through a group of nonlinear partial differential equations. For very long plates with both ends clamped, the buckling mode is composed of a series of periodically repeating buckling waves and hence an infinite plate model with only one buckle wave is effective to predict the buckling behaviour. The infinite orthotropic plate is further simplified to a one-dimensional mechanical model by assuming a lateral buckling mode function. After solving the governing equations of the one-dimensional model in both contact and non-contact regions, shear buckling coefficients of the system and the related buckling modes are obtained. Fitted formulae for the contact shear buckling coefficients in terms of relative foundation stiffness and skin profile parameters are developed. The analytical solutions are verified through examining extreme cases from previous studies and a series of finite element (FE) models in ABAQUS. Finally, a practical example is given to show the efficiency of the developed method.

1. Introduction

Due to their superior performance to flat plates, corrugated sheets with various cross-section shapes, e.g. triangular, trapezoidal, rectangular and dovetailed profile (Fig. 1) are used in many industrial applications including building construction, marine, aerospace and automobile engineering (Rao, 1987; Wright and Hossain, 1997). The corrugated products are sometimes used together with concrete, particularly in civil engineering applications. Although the profiled plates have better characteristics than flat ones, a skin buckling phenomenon occurs frequently when the sheet is subject to compressive or shear loading. Due to the restraints from filler materials, the skin sheets in composite components can only buckle outward, with other regions maintaining contact with the filler. This phenomenon is named unilateral contact buckling, or contact buckling, noting the difference from non-constrained buckling, or bilateral buckling. To study contact buckling problems in composite structures, the influence of filler materials need to be considered.

In practice, filler materials are usually modelled as tensionless foundations. Research on this topic started in the 1950s. Seide (1958) presented a compressive buckling model of an infinite flat plate resting on tensionless elastic foundations in 1958. Since then, contact buckling problems have been investigated by many researchers. The majority of the studies on contact buckling of thin plates have focused on compressive loadings. Wright (1993) studied the local buckling of an orthotropic steel plate constrained by a rigid medium. Uy and Bradford (1996) analysed the elastic local buckling of a steel plate resting on a rigid medium by using a finite strip model. Shahwan and Waas presented elastic contact buckling models of infinite long plates (Shahwan and Waas, 1991, 1998) and finite rectangular plates (Shahwan and Waas, 1994). Ma et al. conducted a series of investigations on contact buckling in composite members, including compressive unilateral contact buckling between delaminated plates (Ma et al., 2007); local buckling of an infinite flat plate on an elastic tensionless foundation (Ma et al., 2008b); the effects of foundation stiffness and aspect ratio on local buckling of thin plates (Ma et al., 2008a); contact buckling of thin infinite profiled sheets unilaterally restricted by rigid foundations (Ma et al., 2008c) and using both experimental and FE methods to analyse the unilateral contact buckling of thin steel skin

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* Corresponding author.
E-mail address: xing.ma@unisa.edu.au (X. Ma).

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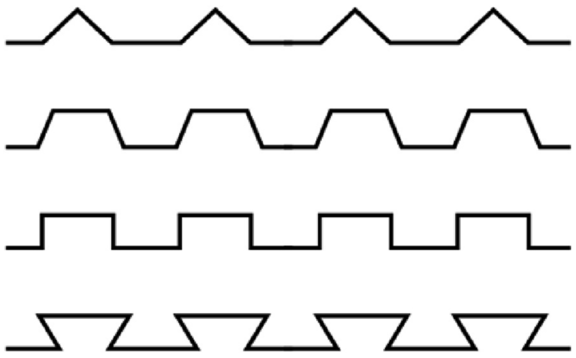

Fig. 1. Cross-section of some corrugated sheets.

on rigid foundations (Keage et al., 2011). In more recent studies, Muradova and Stavroulakis (2012) investigated the unilateral contact buckling of a von Kármán's plate resting on a nonlinear elastic tensionless foundation. Vaseghi et al. (2013) studied the unilateral buckling of laminated composite orthotropic plates constrained by a one-sided foundation. In addition to compression, combined loading effects have also been studied. Smith et al. (1999b,c) and Bradford et al. (2000) used both theoretical and experimental methods to investigate the unilateral buckling behaviour of a rectangular plate restricted by rigid medium and subjected to combined loadings of shear, bending and compression.

By comparison, the study of the shear contact buckling problem is quite limited. Smith et al. (1999a,d) analysed the unilateral and bilateral buckling problem of finite plates restricted on a rigid concrete medium under in-plane pure shear by using the Rayleigh–Ritz method. In a further study (Arabzade et al., 2011), the effect of bolts was taken into account and the skin buckling behaviour of a composite shear wall was analysed using the Rayleigh–Ritz method and the shear buckling coefficients in terms of bolt numbers and aspect ratio were obtained. Ma et al. (2008c) investigated the unilateral buckling behaviour of profiled skin sheet on rigid foundations under in-plane shear. The profiled skin sheet was modelled as an orthotropic plate and the analytical and fitted solutions of the buckling coefficient were obtained. The investigation of the contact buckling behaviour of an infinite thin plate subjected to in-plane shear loading was also presented (Ma et al., 2011).

To date, two types of shear buckling problems have been solved successfully, namely, flat plates on tensionless rigid or elastic foundations, and profiled plates on rigid foundations. However, the shear buckling behaviours of profiled plates resting on tensionless elastic foundations have not been adequately studied. In this paper, the contact buckling behaviour of thin profiled sheets constrained by an elastic tensionless foundation (Winkler foundation) under uniform in-plane shear loadings was investigated and the influence of foundation stiffness was studied. The analytical solution and fitted formulae for contact buckling coefficients of profiled sheets subject to shear loadings were obtained. Finally, numerical simulations based on the proposed analytical method are presented and compared with FE methods.

2. Analytical solutions to contact buckling problems under shear

2.1. Governing equations of a buckling profiled sheet

In this study, the buckling behaviour of an infinite corrugated skin sheet constrained by a tensionless elastic filler and subjected to in-plane shear loading was investigated (Fig. 2(a)). Only plates with clamped edges were taken into account for examples. The filler material was simplified as tensionless Winkler foundations (springs) (Fig. 2(b)). The unknown non-contact and contact areas between the sheet and foundation are illustrated in local coordinate systems (x_1,y,w_1) and (x_2,y,w_2), respectively (Fig. 2(c)).

The buckling problem of the profiled sheet in Fig. 2(c), can be described through the following governing equation

$$D_x w_{i,x_ix_ix_ix_i} + D_{xy} w_{i,x_ix_iyy} + D_y w_{i,yyyy} + 2N_{xy} w_{i,x_iy} = q_i \quad |x_i| \le a_i/2 \tag{1}$$

The stiffness of the corrugated plate may be determined as (Ma et al., 2008c; Samanta and Mukhopadhyay, 1999)

$$D_x = \frac{E^s I_{11}}{b} \tag{2a}$$

$$D_y = \frac{E^s b}{12s} \tag{2b}$$

$$D_{xy} = \frac{Gt^3 s}{3b} = \frac{E^s t^3 s}{6(1+\nu^s)b} \tag{2c}$$

where E^s, b, t, ν^s are the elastic modulus, width, thickness and Poisson's ratio of the skin sheet, respectively. s is the arc length measured along the profiled cross section (Fig 2(a)). I_{11} is the second moment of area of the profiled cross-section about its neutral axis. The shear force N_{xy} is

$$N_{xy} = \tau_{xy} t \tag{3}$$

where τ_{xy} is the shear stress.

$$q_1 = 0, \quad |x_1| \le a_1/2 \quad \textit{for non-contact zone} \tag{4a}$$

$$q_2 = q_2(x_2, y) \quad |x_2| \le a_2/2 \quad \textit{for contact zone} \tag{4b}$$

Assuming the following equations,

$$\mathrm{r} = \frac{s^2}{b^2}\frac{1}{1+\nu^s} - 1 \tag{5}$$

$$K = \frac{b^2 t \tau_{xy}}{\pi^2 D_y} \tag{6}$$

Thus, Eq. (1) may be rewritten as Eq. (7)

$$\frac{D_x}{D_y} w_{i,x_ix_ix_ix_i} + 2(1+r) w_{i,x_ix_iyy} + w_{i,yyyy} + 2\frac{\pi^2 K}{b^2} w_{i,x_iy} = \frac{q_i}{D_y} \quad |x_i| \le a_i/2 \tag{7}$$

Assuming $w_i(x_i, y) = f_i(\overline{x}_i) g(y)$ and the filler material as a Winkler foundation, Eq. (4) can be written as,

$$q_i(x_i, y) = -k_i f_i(\overline{x}_i) g(y) \tag{8}$$

where $i = 1$ and 2, f_i and g are the x-axis (longitudinal direction) and the y-axis (lateral direction) buckling mode function, respectively; k_1 (equal to 0) is the stiffness factor of the filler in non-contact areas and k_2 is the stiffness factor of the filler in contact areas. Due to $k_1 = 0$, the contact force q_1 is 0 in the non-contact areas (the skin sheet moves outward), while the value of the contact force q_2 is $k_2 f_2 g$ for the contact areas.