European Journal of Mechanics A/Solids 58 (2016) 221–232

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Instrumented indentation of a non-equal biaxial prestretched hyperelastic substrate

V.I. Zafiropoulou [a], Th. Zisis [b], A.E. Giannakopoulos [a, *]

[a] Laboratory of Strength of Materials and Micromechanics, Department of Civil Engineering University of Thessaly, Volos, 38336, Greece
[b] Mechanics Division, National Technical University of Athens, 5 Heroes of Polytechnion Ave., Zographou, GR-15773, Greece

ARTICLE INFO

Article history:
Received 27 November 2015
Accepted 29 January 2016
Available online 9 February 2016

Keywords:
Indentation
Hyperelasticity
Prestretching
Property extraction

ABSTRACT

The present work investigates the instrumented indentation of a prestretched hyperelastic substrate. The substrate is incompressible and has an elastic energy density which is isotropic in the reference frame. The analysis focuses on spherical indentation, but the results can be extended to all axisymmetric indenters. The substrate is prestretched biaxially, but it is not necessarily equal-biaxial. Due to the non-equal biaxial stretching, the contact region may obtain an elliptical form with principal axes along the stretching directions with the larger axis along the smallest surface stretch. The problem resembles that of the linear contact of an orthotropic substrate; however the problem under investigation is different. Based on the theoretical analysis we propose a methodology that can be used to solve the inverse problem of material characterization using instrumented indentation of a prestretched surface when the surface stretching is not equal-biaxial. Our analysis is supported by a set of finite element calculations and experiments.

1. Introduction

Instrumented indentation analysis of hyperelastic prestretched substrates by axisymmetric indenters has been investigated in the past, with the main focus on equal-biaxial prestretching, see for example Green et al. (1952). Furthermore, finite element analysis of such problems has been undertaken by Gent and Yeoh, (2006), Karduna et al. (1997) and Zisis et al. (2015) to mention few.

Turning the attention, to the more general problem of indentation of hyperelastic substrates that are biaxially prestreched (but not necessarily equal-biaxial) one can find only a limited number of analytical studies such as those of Borodish (1990), Filippova (1978) and Gay (2000), which, refer to approximate solutions and take into account only limited hyperelastic or hypoelastic material models (linear elastic and Neo-Hookean). Regarding the numerical approaches on problems of the same nature, we should mention the work of Jabareen et al. (2012) who presented a Finite Element (FE) analysis of spherical contact of a hyperelastic not equal-biaxial prestretched substrate. Finally, the experimental attempts are also very limited, and the work of Barquins et al. (1976) should be noted.

It is within the scope of the present study to present analytical solutions for hyperelastic substrates that are biaxially prestretched in a general manner and subsequently indented by rigid spherical indenters. The proposed solutions are confirmed by extensive FE modeling and experiments on uniaxially stretched rubber materials. Our results can be useful for the extraction of material properties through instrumented indentation tests, due to the fact that prestretching can circumvent the uniqueness issue of the inversion problem that is inherent in instrumented indentation problems, as we have already shown for the equal-biaxial prestretching, Zisis et al. (2015).

The novelty of this work lies on an important observation: although initially isotropic, the prestretch alters the substrate to an anisotropic solid when it comes to be subsequently indented. In this way, the indentation response creates opportunities to determine the material constants. Axisymmetric rigid indenters indenting anisotropic substrates are known to create elliptical contact areas. Elliptical contact regions of a rigid sphere pressed on the surface of an anisotropic substrate have been obtained in the context of linear elasticity due to the anisotropic elastic constants of the substrate, Willis (1966); while important

* Corresponding author. Tel.: +30 24210 74179.
E-mail address: agiannak@civ.uth.gr (A.E. Giannakopoulos).

http://dx.doi.org/10.1016/j.euromechsol.2016.01.016
0997-7538/

contributions for contact of orthotropic and transversely isotropic substrates are those of Sveklo (1974), Ovaert (1993) and Shi et al. (2003).

It is of interest to elaborate on the approximate methods that have been proposed in the past in order to solve contact problems of anisotropic elastic materials. Of particular importance are the works of Menditto et al. (1993), Swadener and Pharr (2001) and Vlassak et al. (2003). Menditto et al. (1993) proposed an approximate method based on the mapping technique, initially stated by Lodge (1952) and (1955), which can transform known results of isotropic linear elasticity. The same approach was followed by Arimitsu et al. (1994), Nemish (2000) and Ostrosablin (2006).

A different line of approach can be the consideration of the prestretching of the substrate as a remap of the axisymmetric indenter to a non-axisymmetric one, Gay (2000). Indeed, a spherical indenter could be mapped to an ellipsoid indenter which can produce elliptic contact areas, as predicted in the context of linear elasticity by Johnson (1985). This model is interesting, nevertheless, this approach implies a hypoelastic material response which may not be suitable for many materials that are more appropriately modeled as hyperelastic.

It is clear that Biot's method predicts that prestretching will produce an incrementally linear model that is elastically orthotropic, due to the different prestretches. This means that one can utilize the available methodology related to the extraction of the elastic properties of anisotropic materials by means of instrumented indentation. A central idea that we explore in this work is the connection between eccentricity of the elliptical contact area and the overall contact response. Another key idea is the incorporation of the closed form results that are available in the literature for the case of an equal-biaxial prestretching by taking the equal-biaxial part of the prestretch as the reference configuration in the incremental deformation method. The theoretical work is based on the following points:

a) The application of the prestretch “transforms” the initially isotropic substrate to an orthotropic solid. The incremental contact problem resembles (but is not identical) that of a linear elastic analysis of an orthotropic substrate.
b) The subsequent (incremental) contact problem with a rigid axisymmetric indenter (a sphere in this case) creates an elliptical contact area which has an eccentricity that relates to the ratio of the principal stretches. The anisotropic equivalence applies as long as the incremental problem does not loose ellipticity.
c) There is a connection between eccentricity of the contact area and the hyperelastic model.
d) The surface biaxial stretches decouple to an equal-biaxial and a shear-like part, where the equal-biaxial part has already been investigated by the present authors. The shear-like part is responsible for the eccentricity of the contact cite.

The paper is structured as follows. We present some general approximate solutions using the incremental analysis of Biot (1965) and the analytic results are compared with extensive numerical analysis (FE) of a Mooney-Rivlin model. In the process of the analysis, we review some approximate solutions that have been already presented in the literature and are of utmost importance in this work. Finally, we present an experimental procedure for uniaxially prestretched rubber specimens that are subsequently indented by spherical steel indenters. We verify the results of the present methodology by applying it to specimens that we already know the material properties for comparison.

2. Material models and constraints of properties

In order for the anisotropic equivalence to be used for the subsequent indentation problem that follows prestretch, the ellipticity conditions must be satisfied. We must therefore make sure that possible instabilities are avoided – that is, the elasticity tensor is positive definite and possible surface wrinkling is avoided. Assuming homogeneity, incompressibility and material isotropy, the elastic potential of the hyperelastic material is of the form

$$W = W(I_1,\ I_2), \tag{1}$$

where W is a function of the fundamental scalar invariants in terms of the principal stretches of the deformation $\lambda_i>0$:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \tag{2}$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2, \tag{3}$$

with the incompressibility condition $\lambda_1\lambda_2\lambda_3 = 1$.

The principal Cauchy (*true*) stresses are

$$\sigma_i = -p + 2\lambda_i^2\left(W_1 + \left(I_1 - \lambda_i^2\right)W_2\right), \tag{4}$$

where p is the pressure and

$$W_a = \frac{\partial W}{\partial I_a}, \quad W_{a\beta} = \frac{\partial^2 W}{\partial I_a \partial I_\beta}, \quad a,\ \beta = 1, 2. \tag{5}$$

The shear modulus of the material at infinitesimal deformations can be written as

$$G_0 = 2(W_1(3,3) + W_2(3,3)), \tag{6}$$

with the assumption that $G_0 > 0$. Accordingly, the infinitesimal elastic modulus is $E_0 = 3G_0$.

Criscione et al. (2000) presented an invariant basis for natural strain which yields orthogonal stress response terms in isotropic hyperelasticity, in order to obtain the best loading conditions for material property extraction. They suggest that equal-biaxial stretch state and uniaxial stretching λ_1 are equally good alternatives for experimentation with similar natural strain invariant $K_2 = \sqrt{3/2}\ln(\lambda_1)$, in their notation.

The general ellipticity conditions for incompressible isotropic hyperelastic materials where established by Zee and Sternberg (1983). When the local principal stretches are associated with an axisymmetric state or a uniaxial stretch as

$$\lambda_1 = \lambda_2 = \lambda,\ \ \lambda_3 = \lambda^{-2}, \tag{7}$$

$$I_1 = 2\lambda^2 + \lambda^{-4},\ \ I_2 = 2\lambda^{-2} + \lambda^4, \tag{8}$$

then, the necessary and sufficient ellipticity conditions read

$$W_1 + \lambda^{-4}W^4 > 0, \tag{9}$$

$$\lambda^4\left(W_1 + \lambda^2 W_2\right) + 2\left(\lambda^3 - 1\right)^2\left(W_{11} + \lambda^2 W_{12} + \lambda^4 W_{22}\right) > 0. \tag{10}$$

When $W = W(I_1)$, the general ellipticity conditions are

$$W_1 > 0, \tag{11}$$