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On chiral effects in strain gradient elasticity

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ABSTRACT

This paper is concerned with the problem of uniformly loaded bars in strain gradient elasticity. We study the deformation of an isotropic chiral bar subjected to body forces, to tractions on the lateral surface and to resultant forces and moments on the ends. Examples of chiral materials include some auxetic materials, bones, some honeycomb structures, as well as composites with inclusions. The three-dimensional problem is reduced to the study of some generalized plane strain problems. The method is used to study the deformation of a uniformly loaded circular cylinder. New chiral effects are presented. The flexure of a chiral cylinder, in contrast with the case of achiral materials, is accompanied by extension and bending. The salient feature of the solution is that a uniform pressure acting on the lateral surface of a chiral circular elastic cylinder produces a twist around its axis.

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1. Introduction

The mechanical behaviour of chiral materials is of interest for the investigation of carbon nanotubes (see, e.g., Wang and Wang, 2007; Chandraseker et al., 2009; Askes and Aifantis, 2009; Zhang et al., 2010), auxetic materials (Spadoni and Ruzzene, 2012) and bones (Lakes et al., 1983; Park and Lakes, 1986). The chiral effects cannot be described within classical elasticity (Lakes, 2001). The strain gradient theory of elasticity (Toupin, 1962; Mindlin, 1964; Mindlin and Eshel, 1968) is adequate to describe the deformation of chiral elastic solids (Papanicolopulos, 2011 and references therein).

In this paper we study the deformation of a homogeneous and isotropic chiral bar in the framework of the strain gradient elasticity. Mindlin (1964) presented three forms of the strain gradient elasticity. The relations among the three forms have been presented by Mindlin and Eshel (1968). Throughout this paper we will use the first form of the strain gradient elasticity. The three forms of the theory lead to the same displacement equations of motion for isotropic solids. The constitutive equations of isotropic chiral elastic solids in the strain gradient theory of elasticity have been established by Papanicolopulos (2011). In the present paper we consider the equilibrium of a right cylinder which is subjected to tractions on the lateral surface, to body forces, and to resultants forces and moments on the ends. We assume that the body forces and the lateral tractions are independent of the axial coordinate. The three-dimensional problem is reduced to the study of some two-dimensional problems. The paper is structured as follows. First, we present the basic equations of isotropic chiral elastic solids and formulate the problem of uniformly loaded cylinders. Then, we define the generalized plane strain problem and introduce four auxiliary plane problems necessary to investigate the deformation of loaded cylinders. In the following section we establish the solution of the problem of uniformly loaded bar. In the classical elasticity this problem is known as Almansi–Michell problem and was studied in various works (Khatiashvili, 1983; Ieşan and Quintanilla, 2007; Ieşan, 2009). We show that the body forces and the tractions on the lateral surface produce extension, torsion, flexure, bending by terminal couples and a plane strain. The solution is used to study the deformation of a circular cylinder. We present new chiral effects. It is shown that the flexure of a chiral cylinder, in contrast with the case of achiral bars, is accompanied by extension and bending. The salient feature of the solution is that a uniform pressure acting on the lateral surface of a chiral circular cylinder produces a twist around its axis.

2. Basic equations

In this section we present the basic equation of isotropic chiral elastic solids in the first strain-gradient theory and the formulation of the problem of uniformly loaded cylinders. We consider a body

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that in undeformed state occupies the region B of euclidean three-dimensional space and is bounded by the surface ∂B. We refer the deformation of the body to a fixed system of rectangular axes Ox_k, ($k = 1,2,3$). Let $\mathbf{n}$ be the outward unit normal of ∂B. Letters in boldface stand for tensors of an order $p \geq 1$, and if $\boldsymbol{v}$ has the order p, we write $v_{ij\ldots k}$ (p subscripts) for the components of $\boldsymbol{v}$ in the Cartesian coordinate system. We shall employ the usual summation and differentiation conventions: Latin subscripts (unless otherwise specified) are understood to range over the integers (1,2,3), whereas Greek subscripts to the range (1,2), summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate.

We assume that B is a bounded region with Lipschitz boundary ∂B, consisting of a finite number of smooth surfaces. Let Γ_p be the intersection of two adjoined smooth surfaces and $C = \cup \Gamma_p$. We assume that B is occupied by a homogeneous and isotropic chiral elastic solid. Let $\boldsymbol{u}$ be the displacement vector field. The strain measures are given by (Mindlin and Eshel, 1968)

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \kappa_{ijk} = u_{k,ij}. \tag{1}$$

The constitutive equations for isotropic chiral elastic solids are (Mindlin and Eshel, 1968; Papanicolopulos, 2011).

$$\begin{aligned}
\tau_{ij} &= \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + f\left(\varepsilon_{ikm}\kappa_{jkm} + \varepsilon_{jkm}\kappa_{ikm}\right),\\
\mu_{ijk} &= \frac{1}{2}\alpha_1\left(\kappa_{rri}\delta_{jk} + 2\kappa_{krr}\delta_{ij} + \kappa_{rrj}\delta_{ik}\right)\\
&\quad + \alpha_2\left(\kappa_{irr}\delta_{jk} + \kappa_{jrr}\delta_{ik}\right) + 2\alpha_3\kappa_{rrk}\delta_{ij}\\
&\quad + 2\alpha_4\kappa_{ijk} + \alpha_5\left(\kappa_{kji} + \kappa_{kij}\right) + f\left(\varepsilon_{iks}e_{js} + \varepsilon_{jks}e_{is}\right),
\end{aligned} \tag{2}$$

where τ_{ij} is the stress tensor, μ_{ijk} is the double stress tensor, δ_{ij} is the Kronecker delta, ε_{ijk} is the alternating symbol and $\lambda,\mu,\alpha_s,(s = 1,2,\ldots,5)$, and f are constitutive constants. The terms from (2) which contain the coefficient f represent the chiral part of the constitutive equations. The constitutive equations for chiral solids have been established by Papanicolopulos (2011). In the case of a centrosymmetric (achiral) material the coefficient f is equal to zero. The equilibrium equations are

$$\tau_{ji,j} - \mu_{sji,sj} + F_i = 0, \tag{3}$$

where F_i is the body force per unit volume. Following Toupin (1962) and Mindlin (1964) we introduce the functions P_i,R_i and Q_i by

$$\begin{aligned}
&P_i = (\tau_{ki} - \mu_{rki,r})n_k - D_j(n_r\mu_{rji}) + (D_k n_k)n_s n_p \mu_{spi},\\
&R_i = \mu_{rsi}n_r n_s, \; Q_i = \langle \mu_{pji}n_p n_q \rangle \varepsilon_{jrq}s_r,
\end{aligned} \tag{4}$$

where D_i are the components of the surface gradient, $D_i = (\delta_{ik} - n_i n_k)\partial/\partial x_k$, s_k are the components of the unit vector tangent to C, and $\langle g \rangle$ denotes the difference of limits of g from both sides of C. We denote by $\overline{B}$ the closure of B. We say that the vector field u_j is an admissible displacement field on B provided $u_j \in C^4(B)$. An admissible system of stresses on $\overline{B}$ is an ordered array of functions (τ_{ij},μ_{pqr}) with the following properties: (i) $\tau_{ij} \in C^1(\overline{B}),\ \mu_{ijk} \in C^2(\overline{B})$; (ii) $\tau_{ij} = \tau_{ji}$, $\mu_{ijk} = \mu_{jik}$. By an admissible state on $\overline{B}$ we mean an ordered array of fields $A = (u_i,e_{ij},\kappa_{ijk},\tau_{ij},\mu_{ijk})$ with the properties: (i) u_i is an admissible displacement field on $\overline{B}$; (ii) $e_{ij} \in C^1(\overline{B}),\ \kappa_{ijk} \in C^2(\overline{B}),\ e_{ij} = e_{ji},\ \kappa_{ijk} = \kappa_{jik}$; (iii) (τ_{ij},μ_{ijk}) is an admissible system of stresses on $\overline{B}$. By an external data system on $\overline{B}$ we mean an ordered array $L = (F_i, \tilde{P}_i, \tilde{R}_i, \tilde{Q}_i)$ with the properties: (i) F_i is continuous on $\overline{B}$; (ii) $\tilde{P}_i$ and $\tilde{R}_i$ are piecewise regular on ∂B; (iii) $\tilde{Q}_i$ is piecewise regular on C. We say that $A = (u_i,e_{ij},\kappa_{ijk},\tau_{ij},\mu_{ijk})$ is an elastic state corresponding to the body force F_k if A is an admissible state that satisfies the Equations (1)–(3) on B. The traction problem of elastostatics consists in finding an elastic state that corresponds to the body force F_i and satisfies the boundary conditions

$$P_i = \tilde{P}_i, \; R_i = \tilde{R}_i \text{ on } \partial B \backslash C, \; Q_i = \tilde{Q}_i \text{ on } C, \tag{5}$$

where $\tilde{P}_i, \tilde{R}_i$ and $\tilde{Q}_i$ are prescribed functions.

The potential energy density for isotropic chiral materials is given by

$$\begin{aligned}
W = &\frac{1}{2}\lambda e_{rr}e_{jj} + \mu e_{ij}e_{ij} + \alpha_1\kappa_{iik}\kappa_{kjj} + \alpha_2\kappa_{ijj}\kappa_{irr} + \alpha_3\kappa_{iir}\kappa_{jjr}\\
&+ \alpha_4\kappa_{ijk}\kappa_{ijk} + \alpha_5\kappa_{ijk}\kappa_{kji} + 2f\varepsilon_{ikm}e_{ij}\kappa_{kjm}.
\end{aligned} \tag{6}$$

In what follows we assume that the elastic potential is a positive definite quadratic form in the variables e_{ij} and κ_{ijk}. The restrictions imposed by this assumption on the constitutive coefficients have been presented by Mindlin and Eshel (1968) and Papanicolopulos (2011). The necessary and sufficient conditions for the existence of a solution of the traction problem are (Hlavacek and Hlavacek, 1969)

$$\begin{aligned}
&\int_B F_i dv + \int_{\partial B} \tilde{P}_i da + \int_C \tilde{Q}_i ds = 0,\\
&\int_B \varepsilon_{ijk}x_j F_k dv + \int_{\partial B} \varepsilon_{ijk}\left(x_j\tilde{P}_k + n_j\tilde{R}_k\right)da + \int_C \varepsilon_{ijk}x_j\tilde{Q}_k ds = 0.
\end{aligned} \tag{7}$$

We assume that the region B from here on refers to the interior of a right cylinder of length h with the cross-section Σ and the lateral boundary Π. Let Γ be the boundary of Σ. The Cartesian coordinate frame is supposed to be chosen in such a way that x_3-axis is parallel to the generators of B and the x_1Ox_2 plane contains one of terminal cross-sections. We denote by Σ_1 and Σ_2, respectively, the cross-section located at $x_3 = 0$ and $x_3 = h$. We denote by Γ_α the boundary of the cross-section Σ_α. We assume that the lateral surface Π is smooth, so that Q_i is equal to zero on Π. We shall use Saint-Venant's approach of the problem of elastic cylinders which is based on a relaxed statement in which the pointwise assignment of the terminal tractions is replaced by prescribing the corresponding resultant force and resultant moment. We assume that the cylinder is subjected to body forces, to tractions on the lateral surface and to appropriate global conditions on the ends. The conditions on the lateral boundary are

$$P_i = \tilde{P}_i, \; R_i = \tilde{R}_i \text{ on } \Pi. \tag{8}$$

Let $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3)$ and $\mathcal{M} = (M_1, M_2, M_3)$ be prescribed vectors representing the resultant force and the resultant moment about O of the tractions acting on Σ_1. On Σ_2 there are tractions applied so as to satisfy the equilibrium conditions of the body. For the end located at $x_3 = 0$ we have the following conditions

$$\int_{\Sigma_1} P_\alpha da + \int_{\Gamma_1} Q_\alpha ds = \mathcal{F}_\alpha, \tag{9}$$

$$\int_{\Sigma_1} P_3 da + \int_{\Gamma_1} Q_3 ds = \mathcal{F}_3, \tag{10}$$

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