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Kinematic accuracy analysis of planar mechanisms with clearance involving random and epistemic uncertainty

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ABSTRACT

A general methodology for kinematic accuracy analysis of mechanisms with clearance, involving random and epistemic uncertainty, is presented. A continuous contact force model with consideration of the energy dissipation is applied to evaluate the contact force. Probability space and interval are used to characterize random uncertainty and epistemic uncertainty, respectively. Considering the inefficiency of uncertainty prediction, surrogate models based on neural network approach are constructed. The precision and reliability of the presented method have been validated by numerical simulation. It indicates that, comparing with probability theory, the reliability of the results obtained by the presented method is improved as well as the prediction accuracy, comparing with interval analysis.

1. Introduction

Clearances in mechanisms are inevitable, mainly due to manufacture errors, design tolerance and wear, which may affect position accuracy of practical mechanisms. Thus, many researchers have investigated the effects of joint clearance on kinematic and dynamic behaviors of mechanisms by theoretical and experimental approaches.

Flores and Ambrósio (2004), Flores et al. (2008), Flores and Lankarani (2010) have systematically investigated dynamic behaviors of rigid multibody systems with dry clearance joint and lubricated revolute joint, respectively. Zhao and Bai (2011) focused on the investigation of space robot manipulator with revolute joint clearance. They found that revolute joint clearance seriously affects angular acceleration of the space robot manipulator. Flores and Lankarani (2012) studied the effect of joint clearance on the dynamic behaviors of multibody systems with multiple clearance joints. Chunmei et al. (2002) investigated the effect of joint clearance on a flexible four-bar mechanism. Tian et al. (2010) presented a novel methodology to analyze the effect of revolute clearance joint on dynamic behaviors of planar flexible multibody systems, which is established based on absolute nodal coordinate formulation (ANCF). Then, this method was developed to investigate spatial flexible multibody systems with lubricated cylindrical joints (Tian et al., 2011). Following, a novel method to study the coupling dynamics of a geared multibody system with ElastoHydroDynamic lubricated cylindrical joints was developed (Tian et al., 2015). Machado et al. (2012a) have investigated the effect of clearance joint parameters on the dynamic response of a slider-crank mechanism with lubricated revolute joints. Erkaya (2012) has analyzed kinematics and dynamics of a welding robot manipulator with joint clearance. The results show that joint clearance makes control precision of the manipulators worse. Based on the ANCF and the natural coordinate formulation, Liu et al. (2012) presented a combined control scheme, which combines feedforward torque control and PID controller, to track the desired trajectory for a rigid-flexible multibody system with cylindrical joint. They found that the effect of clearance on dynamic behaviors of flexible system is smaller than that of stiff system. Zhang et al. (2014) used Kriging models to predict the dynamic response of a mechanical system with revolute joint clearance.

In many mechanisms with clearance, material and structural parameters are assumed to be perfect. However, abrasion, imperfections and manufacturing errors are unavoidable in actual mechanisms which result in uncertainties in the systems. Considering parameter uncertainty of mechanisms, Yan and Guo (2011) presented a method of kinematic accuracy analysis for flexible planar mechanisms with stochastic parameters. Their simulation results confirmed that joint clearances and stochastic link lengths affect kinematic accuracy of the mechanisms. Chaker et al. (2013)

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proposed a stochastic method to analyze the influence of the manufacturing errors and clearance on the pose error of the 3-RCC spherical parallel manipulator. In addition to random uncertainty, mechanisms also involve epistemic uncertainty with incomplete knowledge of some characteristics (Helton et al., 2008; Hofer et al., 2002). Probability theory (Hacking, 2001) has been used to represent random uncertainty. However, additional mathematical structures such as interval analysis (Jaulin et al., 2001), evidence theory (Shafer, 1976) and fuzzy set theory (Ross, 2004) were presented to treat epistemic uncertainty in systems. Wu et al. (2013) used interval uncertain method to represent epistemic uncertainty for mechanical systems, which only requires bounds of uncertain parameters. More generally, mechanism parameters involve random and epistemic uncertainty. Dempster-Shafer theory, Second-order probabilistic analysis and Probability bounds analysis were introduced as mathematical structures for the representation of random and epistemic uncertainty (Sentz and Ferson, 2011). Sun et al. (2014) presented a novel method to predict dynamic wear volume in a mechanism involving random and interval uncertainty.

In this work, a general method of kinematic accuracy analysis is presented for mechanisms with clearance, involving random and epistemic uncertainty. The contact model in clearance joint is established by a continuous contact force model, in which energy dissipation is considered. Probability space and interval are applied to represent random uncertainty and epistemic uncertainty, respectively. Finally, a slider-crank mechanism with clearance is used as numerical example to verify precision and reliability of the presented method.

2. Model for revolute clearance joints

Clearances are unavoidable in mechanisms which separate the bearing and the journal. No displacement constraints of mechanisms are introduced in the journal-bearing. However, force constraints of mechanisms have been developed during the contact to replace displacement constraints.

Fig. 1 shows a revolute clearance joint, connecting journal (part of body j) and bearing (part of body i). P_i and P_j define the centers of the bearing and the journal, respectively. $\mathbf{r}_i$ and $\mathbf{r}_j$ are the global coordinate vectors of P_i and P_j in the global coordinate frame, respectively. From Fig. 1, the eccentricity vector $\mathbf{e}_{ij}$ connecting the centers of bearing and journal is expressed as

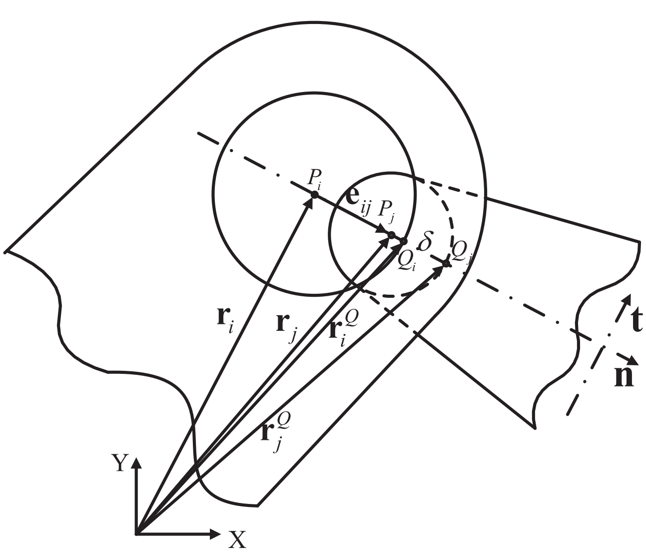

Fig. 1. The model of the revolute clearance joint.

$$\mathbf{e}_{ij} = \mathbf{r}_j - \mathbf{r}_i \tag{1}$$

Thus, the magnitude of the eccentricity vector is represented as follow,

$$\bar{e}_{ij} = \sqrt{\mathbf{e}_{ij}^T \mathbf{e}_{ij}} \tag{2}$$

Then the unit eccentricity vector normal to the surfaces of collision between the journal and the bearing is given by,

$$\mathbf{n} = \frac{\mathbf{e}_{ij}}{\bar{e}_{ij}} \tag{3}$$

The penetration depth due to local deformation between the journal and bearing can be calculated as

$$\delta = \bar{e}_{ij} - c \tag{4}$$

where c is the radial clearance, $c = R_i - R_j$. R_j and R_i are radii of the journal and bearing, respectively.

In Fig. 1, the candidate contact points on body i and body j are Q_i and Q_j, respectively. Then the position of the contact points are given by

$$\mathbf{r}_k^Q = \mathbf{r}_k + R_k \mathbf{n} \quad (k = i, j) \tag{5}$$

The relative normal velocity, v_N, and tangential velocity, v_T, can be expressed as

$$v_N = \left(\dot{\mathbf{r}}_j^Q - \dot{\mathbf{r}}_i^Q \right)^T \mathbf{n} \tag{6}$$

$$v_T = \left(\dot{\mathbf{r}}_j^Q - \dot{\mathbf{r}}_i^Q \right)^T \mathbf{t} \tag{7}$$

where $\dot{\mathbf{r}}_i^Q$ and $\dot{\mathbf{r}}_j^Q$ can be obtained by differentiating (5) with respect to time. The tangent vector $\mathbf{t}$ can be obtained by rotating vector $\mathbf{n}$ by 90° counter clockwise.

When contact between the bearing and journal occurs, an appropriate contact law is introduced as contact forces. The best-known contact model is Hertz contact law (Bhushan, 2002). But the energy dissipation has not been accounted for. Thus, Lankarani–Nikravesh, ESDU-78035 and Dubowsky–Freudenstein contact force models were developed (Lankarani, 1990; Koshy et al., 2013; Askari et al., 2014; Reis et al., 2014) to overcome this problem. The Lankarani–Nikravesh contact force model, in which hysteresis damping is used to represent the energy dissipated for impact, is expressed as follow,

$$F_N = K\delta^n \left(1 + \frac{3\left(1 - c_e^2\right)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right) \tag{8}$$

where $\dot{\delta}$ and $\dot{\delta}^{(-)}$ denote the relative penetration velocity and initial impact velocity, respectively. c_e is the restitution coefficient. The exponent n depends on the materials of the contact surfaces. The generalized stiffness parameter for two spherical surfaces is calculated as (Goldsmith, 1960)

$$K = \frac{4}{3\left(h_i + h_j\right)} \left(\frac{R_i R_j}{R_i - R_j} \right)^{\frac{1}{2}} \tag{9}$$

where R_i and R_j are the bearing and journal radii, the parameters h_i and h_j are evaluated as