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A comparison between two analytical models that approximate notch-root elastic–plastic strain–stress components in two-phase, particle-reinforced, metal matrix composites under multiaxial cyclic loading: experiments

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Abstract

This paper presents the experimental assessment of two simplified models, formulated in a companion paper by the authors, that estimate the elastic–plastic behavior at the notch-root surface of particulate metal matrix composites (PMMCs). The validity of each model is assessed by measuring the notch root strains on circumferentially notched PMMC bars when subjected to a variety of bi-axial cyclic loading paths. The notch root strains are measured using a 3D image correlation system that uses photogrammetric principles and image processing in assessing surface strain fields. Both models are found to be in good agreement with the experimental results in terms of the shape and orientation of the elastic–plastic local shear and axial strains.

Keywords: Particulate metal matrix composites; Image correlation; Notch analysis; Equivalent strain energy density (ESED) method; Neuber's rule

1. Introduction

Remarkable progress has been made in the development and commercial applications of particulate metal matrix composites (PMMCs). However, they are not yet a choice material in many engineering applications that the composite industry desires [1,2]. Researchers [2–5] have indicated that this may be due in part to a lack of experimentally validated models that predict the mechanical behavior of metal matrix composites (MMCs) particularly when they are subjected into multiaxial cyclic loading. In order to properly provide such models, and thus help to exploit the beneficial properties of PMMCs, complexities in modeling external (geometric and loading) and internal (heterogeneous, matrix elastic–plastic behavior) nonlinearities must first be overcome from a theoretical viewpoint. If accomplished, the developed models must be experimentally validated for reliability and thus acceptance.

In [6,7], the authors developed two analytical models to predict the elastic–plastic strain–stress histories at the notch root of PMMC components under multiaxial cyclic loading. These models are based on modified forms of Neuber's rule [8] and the equivalent strain energy density (ESED) [9] method, developed originally for homogeneous materials, the concept of incremental mean field theory [10], and the endochronic theory of plasticity [11]. The preliminary numerical results obtained in [6] indicate that the non-linear behavior at the notch root of the PMMC components tested can be predicted from easy to obtain elastic solutions.

This paper presents an experimental technique based on 3D image correlation system for measuring the notch root strains in PMMC components under a variety of loading conditions. Specifically, this paper aims to measure the elastic–plastic strains at the notch root of PMMCs and to provide validation to the numerical results obtained using the analytical models presented in a companion paper by the authors [6]. Section 2 of this paper gives a summary of the matrix elastic–plastic stress–strain incremental formulations presented in [6]. Section 3 gives a brief description of the experimental equipment. The material description and experimental tests are given in Section 4. In Section 5 the experimental and numerical results obtained are presented and compared for the loading paths tested. Conclusions are drawn in Section 6.

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2. Summary of approximate solutions

Fig. 1 presents a representative notched PMMC component subjected to small increments in loads above the axial, P, and torsional, T, loads shown. The resulting increments in notch-root surface strain and stress components that are to be determined are also shown in the figure. The tensorial equation for the two models (i.e. the modified incremental ESED method and Neuber's rule) presented and expanded in [6] are summarized in this section using the notation shown in Fig. 1. Constitutive relations common to both models are first presented. Relations specific to each model are subsequently presented.

2.1. Elastic–plastic constitutive relations

In [6], the endohronic theory of plasticity [11] was employed, using three terms in the Dirichlet series. The resulting relations, written on the notch-root surface element (see Fig. 1), are given as:

$$\Delta\varepsilon_{11(m)}^{E/N} = \left[\frac{-\upsilon_m}{E_m} + \frac{1}{6}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right] \times \left(\Delta\sigma_{22(m)}^{E/N} + \Delta\sigma_{33(m)}^{E/N}\right) + \frac{1}{2}\sum_{r=1}^{3}\left[\left(C_r\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)^{-1} S_{11}^{E/N(r)}(1-e^{-\alpha_r\Delta z})\Delta z\right], \quad (1)$$

$$\Delta\varepsilon_{22(m)}^{E/N} = \left[\frac{1}{E_m} + \frac{1}{3}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right]\Delta\sigma_{22(m)}^{E/N} + \left[\frac{-\upsilon_m}{E_m} + \frac{1}{6}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right]\Delta\sigma_{33(m)}^{E/N} + \frac{1}{2}\sum_{1}^{3}\left[\left(C_r\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)^{-1} S_{22}^{E/N(r)}(1-e^{-\alpha_r\Delta z})\Delta z\right], \quad (2)$$

$$\Delta\varepsilon_{33(m)}^{E/N} = \left[\frac{1}{E_m} + \frac{1}{3}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right]\Delta\sigma_{33(m)}^{E/N} + \left[\frac{-\upsilon_m}{E_m} + \frac{1}{6}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right]\Delta\sigma_{22(m)}^{E/N} + \frac{1}{2}\sum_{1}^{3}\left[\left(C_r\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)^{-1} S_{33}^{E/N(r)}(1-e^{-\alpha_r\Delta z})\Delta z\right], \quad (3)$$

and

$$\Delta\varepsilon_{23(m)}^{E/N} = \left[\frac{1+\upsilon_m}{E_m} + \frac{1}{2}\left(\sum_{r=1}^{3} C_r\left(\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)\right)^{-1}\Delta z\right]\Delta\sigma_{23(m)}^{E/N} + \frac{1}{2}\sum_{1}^{3}\left[\left(C_r\frac{1-e^{\alpha_r\Delta z}}{\alpha_r}\right)^{-1} S_{23}^{E/N(r)}(1-e^{-\alpha_r\Delta z})\Delta z\right]. \quad (4)$$

In Eqs. (1)–(4), E/N indicates that either ESED or Neuber's elastic–plastic values can be substituted. $\Delta\varepsilon$ and $\Delta\sigma$ are increments in the matrix (m) strain (ε) and stress (σ) respectively, and S represents the deviatoric stress tensor. Furthermore, Δz is the increment in intrinsic time scale, C_r and α_r are material constants, and υ_m and E_m are the matrix Poisson's ratio and elastic modulus, respectively.

2.2. Modified incremental ESED method

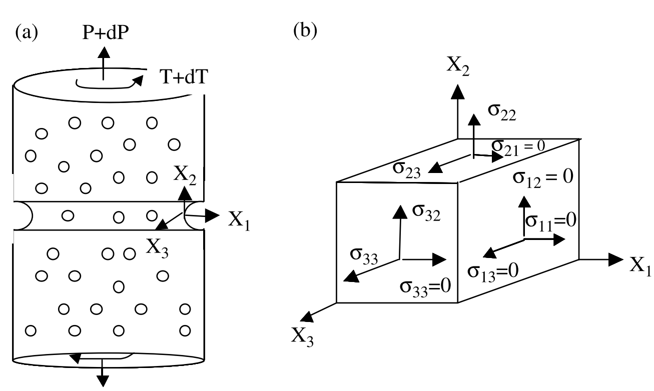
The equivalent strain energy density (ESED) method, originally developed for metals, was reformulated in [6] to incorporate the heterogeneous nature of PMMCs. The resulting relations are given as:

$$\sigma_{22}^{e}\Delta\varepsilon_{22}^{e} = (1-V_f)\left(\sigma_{22(m)}^{E}\Delta\varepsilon_{22(m)}^{E}\right) + (V_f)\left(\sigma_{22(f)}^{e}\Delta\varepsilon_{22(f)}^{e}\right) \quad (5)$$

$$\sigma_{33}^{e}\Delta\varepsilon_{33}^{e} = (1-V_f)\left(\sigma_{33(m)}^{E}\Delta\varepsilon_{33(m)}^{E}\right) + (V_f)\left(\sigma_{33(f)}^{e}\Delta\varepsilon_{33(f)}^{e}\right) \quad (6)$$

$$\sigma_{23}^{e}\Delta\varepsilon_{23}^{e} = (1-V_f)\left(\sigma_{23(m)}^{E}\Delta\varepsilon_{23(m)}^{E}\right) + (V_f)\left(\sigma_{23(f)}^{e}\Delta\varepsilon_{23(f)}^{e}\right) \quad (7)$$

In Eqs. (5)–(7), e stands for the elastic notch root parameters, and E stands for the notch root elastic–plastic



$$d\sigma_{ij} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & d\sigma_{22} & d\sigma_{23} \\ 0 & d\sigma_{32} & d\sigma_{33} \end{vmatrix} \qquad d\varepsilon_{ij} = \begin{vmatrix} d\varepsilon_{11} & 0 & 0 \\ 0 & d\varepsilon_{22} & d\varepsilon_{23} \\ 0 & d\varepsilon_{32} & d\varepsilon_{33} \end{vmatrix}$$

Fig. 1. Problem definition (a) circumferentially notched PMMC bar (b) notch root surface stress state for a given P and T (c) increments in stress and strain for increments dP and dT [1].