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Nonlocal nonlinear free vibration of nanobeams with surface effects

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ABSTRACT

In this paper, nonlinear free vibration analysis of simply-supported nanoscale beams incorporating surface effects, i.e. surface elasticity, surface tension and surface density, is studied using the nonlocal elasticity within the frame work of Euler–Bernoulli beam theory with von kármán type nonlinearity. A linear variation for the component of the bulk stress, σ_{zz}, through the nanobeam thickness is used to satisfy the balance conditions between the nanobeam bulk and its surfaces. An exact analytical solution to the governing equation of motion is presented for natural frequencies of nanobeams using elliptic integrals. The effect of the nanobeam length, thickness to length ratio, mode number, amplitude of deflection to radius of gyration ratio and nonlocal parameter on the normalized natural frequencies of aluminum and silicon nanobeams with positive and negative surface elasticity, respectively, is investigated. It is observed that the surface effects increase natural frequencies of the aluminum nanobeam for all values of the amplitude ratio and the silicon nanobeam at low amplitude ratios while at higher amplitude ratios the surface effects decrease the natural frequencies of the silicon nanobeam. Also, for all values of amplitude ratios, the normalized fundamental natural frequencies of silicon and aluminum nanobeams vary linearly with respect to the nonlocal parameter while this is not the case at higher mode numbers.

1. Introduction

Owing to their high surface-to-volume ratio, nano-scale structures exhibit superior mechanical, electrical and thermal performances than their micro- and/or macro-scale counterparts. In micro/nano electromechanical systems (MEMS/NEMS), they are widely used in many areas, including communications, machinery, information technology, biotechnology and etc. (Evoy et al., 1999; Lavrik et al., 2004). An important phenomenon that has attracted considerable attention in the literature is the size-dependent mechanical behavior of nanobeams due to the surface effects or/and small scale effects.

In classical continuum mechanics, the influences of surface energy are ignored as they are small compared to the bulk energy. For nanoscale materials and structures, however, the surface effects become significant due to the high surface/volume ratio. Both the atomistic simulations and experimental evaluations strongly suggest that the ratio of surface to volume plays a critical role in nano-sized problems. To account for the effect of surfaces/interfaces on mechanical deformation, the surface elasticity theory which considers the surface elasticity, surface stress, and surface density as the surface effects is presented by modeling the surface as a two dimensional membrane adhering to the underlying bulk material without slipping (Gurtin and Murdoch, 1975; Gurtin et al., 1998). Based on this model one important issue is how it is possible to satisfy the balance condition between the nanobeam bulk and its surfaces. To this end, Lu et al. (2006) and Lü et al. (2009) proposed a linear and cubic variation for the normal stress, σ_{zz}, along the thickness of homogenous and functionally graded materials (FGM) nanobeams, respectively. There are many studies considering the surface elasticity theory for wave propagation, buckling, and free linear and nonlinear vibration analyses of nanobeams and CNTs based on different beam theories (Assadi and Farshi, 2011; Fu et al., 2010; Gheshlaghi and Hasheminejad, 2011; Guo and Zhao, 2007; Hosseini-Hashemi et al., 2014a; Hosseini-Hashemi and

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Nazemnezhad, 2013; Hosseini-Hashemi et al., 2014b; Lei et al., 2012; Liu and Rajapakse, 2010; Malekzadeh and Shojaee, 2013; Nazemnezhad and Hosseini-Hashemi, 2014a; Nazemnezhad et al., 2012; Park, 2012; Ren and Zhao, 2004; Song et al., 2011; Wang and Feng, 2007, 2009). However on one hand the surface elasticity theory considers the surface elasticity, surface stress, and surface density as the surface effects, and on the other hand satisfying the balance condition is an important issue in the theory, but most of the studies examine only the surface elasticity and stress effects and there are a few works focusing on the influences of the surface density and satisfying the balance condition as well as the surface elasticity and stress effects (Hosseini-Hashemi et al., 2014a; Hosseini-Hashemi and Nazemnezhad, 2013; Hosseini-Hashemi et al., 2014b; Lu et al., 2006; Nazemnezhad and Hosseini-Hashemi, 2014a; Nazemnezhad et al., 2012).

The nonlocal continuum mechanics (Eringen, 1972, 1983, 2002; Eringen and Edelen, 1972) accounts for the small scale effect by considering the stress state at a given point to be a function of the strain field at all points in the body. Therefore, this continuum theory not only is suitable for modeling submicro- or nano-sized structures, but also avoids enormous computational efforts when compared with discrete atomistic or molecular dynamics simulations. Many researchers have applied the nonlocal elasticity concept for the wave propagation (Wang and Hu, 2005; Wang, 2005), bending, buckling, and vibration (Ece and Aydogdu, 2007; Lim et al., 2010; Maachou et al., 2011; Mohammadi and Ghannadpour, 2011; Nazemnezhad and Hosseini-Hashemi, 2014b; Reddy, 2007, 2010; Wang et al., 2007; Xu, 2006) analyses of beam-like elements in micro- or nano electromechanical systems. For example, Reddy (2007) applied the Eringen nonlocal elastic constitutive relations to derive the equation of motion of various kinds of beam theories (i.e. Euler–Bernoulli, Timoshenko, Reddy and Levinson) and proposed analytical and numerical solutions on static deflections, buckling loads, and natural frequencies of nanobeams.

From literature survey, it can be found that there are few papers in which both surface and small scale effects on static and dynamic behaviors of nanostructures and CNTs are taken into account (Hosseini-Hashemi et al., 2014b; Lee and Chang, 2010; Lei et al., 2012; Malekzadeh and Shojaee, 2013; Wang and Wang, 2011). Hosseini-Hashemi et al. (2014b) considered all parameters of the surface effects on the nonlinear free vibration analysis of simply-supported functionally graded Euler–Bernoulli nanobeams using nonlocal elasticity theory. In addition, they satisfied the balance conditions between FG nanobeam bulk and its surfaces by assuming a cubic variation for the component of the normal stress through the FG nanobeam thickness. The governing equations had solved by using the multiple scales method and only the fundamental natural frequency had reported. In another work, Malekzadeh and Shojaee (2013) studied the surface and nonlocal effects on the nonlinear flexural free vibration of elastically supported non-uniform cross section nanobeams. In the work, only the surface elasticity and the surface stress of the surface elasticity theory were considered and the balance condition was not satisfied. The fundamental natural frequencies were also obtained by using the differential quadrature method (DQM).

The main purpose of the present work is to propose a comprehensive analytical model to study the surface effects, including surface elasticity, surface tension and surface density, on the nonlinear free vibration of nanoscale Euler–Bernoulli beams using nonlocal elasticity. The von Kármán geometric nonlinearity is taken into account with the assumption of linear variation of normal stress through the thickness. The governing equations are derived by using the Hamilton's principle and solved by using the exact solution, elliptic integrals. The surface and small scale effects on the nonlinear natural frequencies of nanobeams are examined for different nanobeam dimensions, vibration amplitudes and thickness ratios.

2. Problem formulation

2.1. Nonlocal elasticity theory

As mentioned earlier, in the nonlocal theory, the stress in a material body point is a function of strain field of the same point and all other ones in material domain; Thus, the stress tensor plays the essential role in this continuum theory which is defined as (Eringen, 2002):

$$t_{ij} = \int_{v} \alpha(|x' - x|)\sigma_{ij}(x')dV' \tag{1}$$

where the volume integral is taken over the body region V; x is the reference point in body which the stress tensor is calculated at any other point like x' in the body; $i, j = x, y, z$ for three dimensional Cartesian coordinate; σ_{ij} is the local stress tensor and $\alpha(|x' - x|)$ is nonlocal kernel function depends on internal characteristic length. Eringen proposed $\alpha(|x' - x|)$ as a Green function of a linear differential operator $\mathscr{L}$ as:

$$\mathscr{L}\alpha(|x' - x|) = \delta(|x' - x|) \tag{2}$$

Substituting Eq. (2) into Eq. (1), the integral forms of nonlocal stress tensor reduces to the differential one as follows:

$$\mathscr{L}t_{ij} = \sigma_{ij} \tag{3}$$

The linear operator is an approximate model of the kernel obtained by matching the Fourier transforms of the kernel in the wave number space with the dispersion curves of lattice dynamics. For curve-fitting at low wave numbers relevant to the small internal length scale Eq. (2) is written as:

$$\left(1 - \varepsilon^2\nabla^2 + \gamma^4\nabla^4 - \ldots\right)t_{ij} = \sigma_{ij}$$

Thus, the linear operator becomes:

$$\mathscr{L} = \left(1 - \varepsilon^2\nabla^2 + \gamma^4\nabla^4 - \ldots\right) \tag{4}$$

Where ε and γ are small parameters proportional to the internal length scale. If first order approximation is to be considered, just the Laplacian form of the operator in Eq. (4) is maintained (Alavinasab, 2009). So for the two-dimensional case:

$$\mathscr{L} = 1 - (e_0 l)^2\nabla^2 \tag{5}$$

In which l is internal length and e_0 is material constant which is defined by the experiment and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian operator.

Equations of motion for nonlocal linear elastic solids are obtained from nonlocal balance law as:

$$t_{ij,j} + f_i = \rho\ddot{u}_i \tag{6}$$

where f_i and u_i are the components of the body force and displacement vector respectively and ρ is the mass density. Using Eq. (3) in Eq. (6) the nonlocal equations of motion in a differential form can be expressed by:

$$\sigma_{ij,j} + \mathscr{L}(f_i - \ddot{u}_i) = 0 \tag{7}$$