European Journal of Mechanics A/Solids 52 (2015) 54–71

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Reflection and transmission of elastic waves at the interface between two gradient-elastic solids with surface energy

Yueqiu Li [a], P.J. Wei [a, b, *], Qihen Tang [b]

[a] Department of Applied Mechanics, University of Sciences and Technology Beijing, Beijing, 100083, China
[b] State Key Laboratory of Nonlinear Mechanics (LNM), Chinese Academy of Science, Beijing

A R T I C L E I N F O

Article history:
Received 11 July 2014
Accepted 9 February 2015
Available online 18 February 2015

Keywords:
Gradient elasticity
Microstructure effects
Surface effects

A B S T R A C T

The reflection and transmission of an incident plane wave at an interface between two gradient elastic solids with surface energy are studied. First, some functions of strain energy density with the surface effects taken into consideration are postulated and the surface effects of gradient elastic solid are incorporated into the constitutive relations. Then, the nontraditional interface conditions by requiring the auxiliary monopolar tractions and the auxiliary dipolar tractions continuous across the interface are used to determine the reflection and transmission coefficients. Some numerical results of the reflection and transmission coefficient in terms of energy flux ratio are given for different microstructure parameters and surface parameters. The microstructure effects and the surface energy effects on the reflection and transmission waves are discussed based on the numerical results. It is found that the microstructure effects result in the appearance of two surface waves and the surface energy effects have apparent different influence on the body waves in gradient elastic solids and the surface waves at the interface. However, this phenomenon becomes pronounced only when the incident wavelength is close to the characteristic length of microstructure.

1. Introduction

It is known that the propagation of elastic waves in the classical elastic solids is not dispersive and the lattice dynamics shows that the lattice waves are dispersive. The inconsistency between them leads to the requirement of modifying the classical elastic theory when it is used to study the propagation of elastic waves with high frequency or short wavelength. The concept of structural level of deformation more and more becomes recognized recently. According to this concept, each mass point of a continuum is regarded as a complex system of interacting structures at a lower structural level. The elastic theory of solids with microstructure in such hypotheses occupies an intermediate position between the classical continuum theory and the lattice theory. In order to take the microstructure effects into consideration, the generalized elastic theories, for example, the nonlocal theory (Eringen, 2001), the micromorphic theory (Eringen and Gortler, 1964), the couple stress theory (Mindlin and Tiersten, 1962; Toupin, 1962), the micropolar theory (Eringen, 1966) and microstretch theory (Eringen, 1990), were proposed successively in the past several decades. Because the degree of freedom of the material particle increases in these generalized continuum theories, the modes of vibration of material particle become more complicated and therefore create many new wave modes which cannot be observed in the classical elastic solids. Graff and Pao (1967) pointed that there are two propagating body waves (one is dispersive and other is not dispersive) and one dispersive non-propagating body wave in the couple-stress solids. Gourgiotis et al. (2013) pointed that there are two dispersive propagating body waves and two evanescent body waves in the gradient elastic solids. Parfitt and Eringen (1969), Tomar and Gogna (1995) and Tomar and Garg (2005) showed that there are four dispersive body waves in a micropolar solid and five dispersive body waves in the microstretch solids. The microstructure effects not only lead to the new body wave modes in the interior of solid but also the P-type and S-type surface waves at the boundary of solids. It is known that the surface of solid exhibits properties quite different from those associated with their interior. The surface stress is present in liquid in the form of surface tension and more generally may be present at boundary of any solids. Shenoy (2005) studied the surface stress of solids by the atomic simulations and

* Corresponding author. Department of Applied Mechanics, University of Sciences and Technology Beijing, Beijing, 100083, China.
E-mail address: weipj@ustb.edu.cn (P.J. Wei).

http://dx.doi.org/10.1016/j.euromechsol.2015.02.001
0997-7538/

demonstrated that the elastic constants of surface can be either positive or negative. Due to the development of nanoscience and nanotechnology, there is an increasing demand for understanding the mechanical behaviours of small-sized materials and structures. At the micro and nano scales, surfaces and interfaces may have significant effects on the mechanical properties of solids due to the increasing ratio of surface/interface area to volume (Duan and et al., 2005; Miller and Shenoy, 2000). Recently, Wang (2007, 2008) studied the diffraction of elastic wave from a nanocylinder and a nanosphere with surface effects. Hasheminejad et al. (Hasheminejad and Avazmohammadi, 2009) and Qiang and Wei (Qiang et al., 2012; Qiang and Wei, 2013) studied the effective propagation constants of elastic wave in random nanocomposite material with the surface effects considered. Liu et al. (2012) and Cai and Wei (2013) further studied the surface/interface effects on band gap of elastic waves in the periodic nanocomposite material. However, in their studies, the surface/interface effects are taken into consideration based on the surface elastic theory proposed by Gurtin and Murdoch (1975). In this surface elastic theory, a surface is regarded as a negligibly thin membrane adhered to the bulk without slipping. The elastic moduli of the thin membrane are different from those of the bulk material. The surface effects are due to the presence of the surface stress in the thin membrane. Ogden and Steigmann (2002) further generalized this theory and proposed that the thin membrane (a coat on the bulk material itself) may have its own bending stiffness. According to this theory, they studied the secular equation of surface wave propagation on a pre-stressed incompressible isotropic elastic half-space with a thin coating on its plane boundary. Recently, a surface model taking the surface mass, surface elasticity and surface inertia into consideration is proposed by Placidi et al. (2013) and dell'Isola et al. (2012). The surface model was used to model an interface between two second gradient elastic solids and lead to four types of connections between the two sides of the interface, namely, the “generalized internal clamp”, the “generalized internal hinge”, the “generalized internal roller” and the “generalized internal free ends”. All these constraints can be assumed to model different types of surface microstructure which results in different types of connection between the two parts of the continuum.

There is an alternative method to consider the surface effects. The concept of a thin membrane (with elasticity, and mass even) without thickness adhered to the bulk material is discarded. The surface effects are incorporated into the constitutive relation of bulk material by the direct postulation of a specific function of strain energy density, for example, Vardoulakis and Georgiadis (1997), Exadaktylos (1999), Tsepoura et al. (2002), Georgiadis et al. (2000), Yerofeyev and Sheshenina (2005) and others. Recently, the propagation of different types of elastic waves in a gradient-elastic medium with surface energy is studied (Vardoulakis and Georgiadis, 1997; Georgiadis et al., 2000; Yerofeyev and Sheshenina, 2005). The dispersion characteristics of longitudinal and shear body waves, Rayleigh surface waves and antiplane shear surface waves, antiplane shear waves in a layer, and torsional surface waves are analysed. However, the surface effects on the reflection and transmission waves at an interface have not reported up to now. In this paper, the reflection and transmission problems of an incident plane wave at an interface between two gradient elastic solids are studied. The surface effects of dipolar gradient elastic solids are considered by the direct postulation of a specific function of strain energy density. The reflection and transmission coefficients in terms of energy flux ratio are calculated and are validated by the energy conservation through a unit area at interface. The microstructure effects and the surface effects on the reflection and transmission waves are discussed based on the numerical results.

2. Some specific functions of strain energy density with surface energy

According to the Mindlin's elastic theory of solids with microstructure, the strain energy density can be expressed as

$$W=\frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl}+\frac{1}{2}b_{ijkl}\gamma_{ij}\gamma_{kl}+\frac{1}{2}a_{ijklmn}\chi_{ijk}\chi_{lmn}+d_{ijklm}\gamma_{ij}\chi_{klm}+f_{ijklm}\chi_{ijk}\varepsilon_{lm}+g_{ijkl}\gamma_{ij}\varepsilon_{kl}, \tag{1}$$

where c_{ijkl},b_{ijkl},a_{ijklmn},d_{ijklm},f_{ijklm} and g_{ijkl} are the components of elastic tensors of materials. ε_{ij}, $\gamma_{ij}(=u_{j,i}-\psi_{ij})$ and $\chi_{ijk}(=\psi_{jk,i})$ are the macro-strain of macro-medium, relative deformation (the difference between the macro-displacement gradient and the micro-deformation) and the micro-deformation gradient (the macro-gradient of the micro-deformation), respectively. If the relative deformation is assumed to be zero, namely, $\gamma_{ij}=0$, then, the strain energy density function is simplified as

$$W=\frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl}+\frac{1}{2}a_{ijklmn}\chi_{ijk}\chi_{lmn}+f_{ijklm}\chi_{ijk}\varepsilon_{lm}, \tag{2}$$

It means that the strain energy density is not only dependent of the strain but also the strain gradient. Even for isotropic medium, there are too many material parameters involved still. Here, a simplified version of Eq. (2) is given as following.

$$W=\left(\frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj}+\mu\varepsilon_{ij}\varepsilon_{ij}\right)+\left(\frac{1}{2}\lambda c\varepsilon_{ii,k}\varepsilon_{jj,k}+\mu c\varepsilon_{ij,k}\varepsilon_{ji,k}\right)+\left[\frac{1}{2}\lambda b_k\left(\varepsilon_{ii}\varepsilon_{jj}\right)_{,k}+\mu b_k\left(\varepsilon_{ij}\varepsilon_{ji}\right)_{,k}\right], \tag{3}$$

where the first term is the contribution from the strains in bulk material; the second term is the contribution from the strain gradient in bulk material. Consider that

$$\int_V b_k\left(\varepsilon_{ij}\varepsilon_{ji}\right)_{,k}\mathrm{d}V=\int_S\left(\varepsilon_{ij}\varepsilon_{ji}\right)b_k n_k\mathrm{d}S, \tag{4}$$

The third term in Eq. (3) is the contribution from the surface material. Let

$$E_1=\frac{1}{2}\lambda b_k n_k\left(\varepsilon_{ii}\varepsilon_{jj}\right)+\mu b_k n_k\left(\varepsilon_{ij}\varepsilon_{ji}\right), \tag{5}$$

then, E_1 is the surface energy of unit surface area and it includes the contribution from the surface normal stress and the surface shear stress, respectively. Define

$$\tau_{ij}=\frac{\partial W}{\partial\varepsilon_{ij}}=\left(\lambda\delta_{ij}\varepsilon_{pp}+2\mu\varepsilon_{ij}\right)+b_k\left(\lambda\delta_{ij}\varepsilon_{pp,k}+2\mu\varepsilon_{ij,k}\right), \tag{6}$$

$$\mu_{kij}=\frac{\partial W}{\partial\chi_{kij}}=b_k\left(\lambda\delta_{ij}\varepsilon_{pp}+2\mu\varepsilon_{ij}\right)+c\left(\lambda\delta_{ij}\varepsilon_{pp,k}+2\mu\varepsilon_{ij,k}\right). \tag{7}$$

where λ and μ are the classical lame constants; c is a microstructure parameter with dimension of m^2; b_k is a surface parameters with dimension of m. τ_{ij} is the Cauchy stress or monopolar stress with the dimensions of Nm^{-2} and μ_{ijk} the dipolar stress with the dimensions of Nm^{-1}. The monopolar stress and the dipolar stress are corresponding with the notion of monopolar force and the dipolar force, respectively. The monopolar force is the classical force and the dipolar forces are the anti-parallel forces acting between the micro-media contained in the continuum with microstructure.