European Journal of Mechanics A/Solids 52 (2015) 85–94

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Axisymmetric vibrations and buckling analysis of functionally graded circular plates via differential transform method

Roshan Lal[*], Neha Ahlawat

Dept. of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India

ARTICLE INFO

Article history:
Received 7 May 2014
Accepted 10 February 2015
Available online 25 February 2015

Keywords:
Differential transform
Buckling and axisymmetric vibrations
Functionally graded circular plates

ABSTRACT

Analysis and numerical results for the axisymmetric vibrations of functionally graded circular plates subjected to uniform in-plane force have been presented on the basis of classical plate theory. The mechanical properties of the plate material are assumed to vary as a general function of thickness parameter. A semi-analytical approach namely, differential transform method has been employed to solve the differential equation governing the motion of such simply supported and clamped plates. The effect of volume fraction index and the in-plane force parameter has been studied on the first three natural frequencies of vibration. By allowing the frequency to approach zero, the critical buckling loads for both the plates have been computed. Two-dimensional mode shapes for specified plates have been plotted. A comparison of results has been presented.

1. Introduction

Since the invention of functionally graded materials (FGMs) in 1984 (Koizumi, 1997, 1993), many researches dealing with vibration characteristics of FGM plates have appeared in the literature due to their wide applications in nuclear energy reactors, solar energy generators, space shuttle etc. and particularly in defence — as penetration resistant materials used for armour plates and bulletproof vests. FGMs are usually made from a mixture of ceramic and metal as they are able to withstand high temperature gradient environments while maintaining their structural integrity, particularly in spacecraft and nuclear plants. Basically, these materials are microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by gradually varying the volume fraction of the constituent materials (Suresh and Mortensen, 1998).

In the recent years, numerous studies on the static/dynamic behaviour of FGM plates/shells of various geometries have been made and reported in the Refs. Jha et al. (2013), Feldman and Aboudi (1997), Abrate (2006), Najafizadeh and Heydari (2007), Li et al. (2008), Saidi et al. (2009), Wirowski (2009), Efraim (2011), Kermani et al. (2012), Shamekhi (2013), Kumar and Lal (2013), Jabbari et al. (2014a), Tornabene (2009), Tornabene and Reddy (2013) and Fantuzzi et al. (2014), to mention a few. Out of these, Ref. Jha et al. (2013) is an excellent review of the work upto 2012, on the deformation, stress, vibration and stability problems of FG plates of various geometries. Feldman and Aboudi (1997) analysed the elastic-bifurcational buckling of FG rectangular plates under in-plane compressive loading employing a combination of micromechanical and structural approach. Abrate (2006) studied the free vibrations, buckling and static deflections of FG plates and presented a comparison with homogeneous isotropic plates. The closed form solutions for the axisymmetric buckling of FG circular plates under uniform radial compression have been obtained by Najafizadeh and Heydari (2007) on the basis of higher order shear deformation plate theory. Li et al. (2008) used stress functions in obtaining the analytical solutions for simply supported and clamped FG circular plates subject to an axisymmetric transverse load. Axisymmetric bending and buckling analysis for thick FG circular plates using unconstrained third-order shear deformation plate theory have been presented by Saidi et al. (2009). In this paper, the solutions have been derived in terms of the corresponding responses of homogeneous circular plates based on the classical plate theory. The free vibrations of thin FGM annular plates have been studied by Wirowski (2009) using finite difference method. An empiric/accurate formula to obtain the natural frequencies of FGM plates from the frequencies of isotropic plates has been derived by Efraim (2011) and the results for annular plates with various boundary conditions has been presented. Very recently, Kermani et al. (2012) presented the free vibration analysis of multi-directional FG circular and annular plates using state space-based

---

[*] Corresponding author. Tel.: +91 1332 285151; fax: +91 1332 273560.
E-mail address: rlatmfma@iitr.ac.in (R. Lal).

http://dx.doi.org/10.1016/j.euromechsol.2015.02.004
0997-7538/

differential quadrature method. The free vibrations of FGM circular plates of variable thickness under axisymmetric condition have been analysed by Shamekhi (2013) using a meshless method in which point interpolation approach is employed for constructing the shape functions for Galerkin weak form formulation. Kumar and Lal (2013) predicted the natural frequencies for axisymmetric vibrations of two-directional FG annular plates resting on Winkler foundation using differential quadrature method and Chebyshev collocation technique. In a significant contribution, analysis and closed form solutions for the buckling of FG thin circular plate made of saturated porous material have been presented by Jabbari et al. (2014a) based upon Love–Kirchhoff plate theory. The above studies have been made assuming one parameter power-law distribution for FGM. However, in the literature few studies have been reported in which a more general four parameter power-law distribution for FGM has been proposed and few of them are reported in Refs. Tornabene (2009), Tornabene and Reddy (2013) and Fantuzzi et al. (2014).

In many practical situations, particularly in the automotive industry and ship buildings, plate type structural components may be subjected to in-plane dynamic loads of different types, which may induce buckling, a phenomenon which is highly undesirable. In this regard, efforts have been made by researchers to analyse the effect of in-plane loads on the vibration characteristics of plates of various geometries and important ones are reported in Refs. Paydar (1990), Thevendran and Wang (1996), Xiang and Wei (2004), Gupta et al. (2006), Hu et al. (2006), Lal and Dhanpati (2007) and Ghaheri et al. (2014). Paydar (1990) used the finite difference method to present the axisymmetric buckling analysis of annular sandwich plate with core of linearly varying thickness. Elastic buckling of thin annular plates elastically restrained against rotation at inner and outer edges has been analysed by Thevendran and Wang (1996) employing Rayleigh–Ritz method. Xiang and Wei (2004) have provided the first known exact solutions using Levy-type solution method and domain decomposition method for the buckling and vibration of stepped rectangular Mindlin plates. In a paper, Gupta et al. (2006) used Ritz method to study the effect of Winkler foundation on the buckling and vibration of polar orthotropic circular plate of linearly varying thickness. Hu et al. (2006) have analysed the finite element buckling of composite laminated skew plates using nonlinear material mode and nonlinear in-plane shear formulation. Buckling and vibration of non-homogeneous rectangular plates with exponentially varying thickness have been studied by Lal and Dhanpati (2007) employing quintic spline technique. Ghaheri et al. (2014) have investigated the buckling and vibration characteristics of symmetrically laminated composite elliptical plate resting on Winkler foundation using Ritz method.

The effect of thermal environment on the free vibrations of FG annular plates with mixed boundary conditions has been analysed by Shi and Dong (2012) employing Chebyshev–Ritz method on the basis of 3D elasticity theory. Recently, thermal buckling of FGM rectangular plates on the basis of classical plate theory for four different types of thermal loading has been presented by Javaheri and Eslami (2002) using Galerkin's method. Shooting method has been used by Ma and Wang (2004) to study the axisymmetric bending and buckling problems of FGM plates using third order shear deformation theory. In this paper, a relationship between third order plate theory for axisymmetric bending and buckling of FGM circular plates and the classical plate theory solution for isotropic plate has been presented. Thermal buckling of circular FGM plates based on higher order shear deformation plate theory has been analysed by Najafizadeh and Heydari (2004) using energy method and Galerkin's formulation. Finite element method has been applied by Prakash and Ganapathi (2006) to analyse the asymmetric thermal buckling and vibration characteristics of FGM circular plates. Element-free kp-Ritz method has been employed by Zhao et al. (2009) to study the mechanical and thermal buckling analysis of FGM plates with arbitrary geometry, including the plates containing circular and square holes at the centre. Zenkour and Sobhy (2011) have investigated the thermal buckling analysis of rectangular FGM plates resting on two-parameter Pasternak foundation using the trigonometric shear deformation plate theory. Very recently, Ghomshei and Abbasi (2013) developed a finite element formulation to analyse the axisymmetric buckling of annular FGM plates of linearly varying thickness subjected to thermal loads. Ansari et al. (2013) presented thermal buckling analysis for a Mindlin rectangular FGM microplate on the basis of strain gradient theory using generalized differential quadrature method. Analytical solution in terms of Bessel functions for steady state three dimensional thermoelasticity of FG circular plates due to axisymmetric loads has been given by Jabbari et al. (2014b).

Except the methodologies used in above investigations, there are various numerical techniques such as Frobenius method (Bodaghi and Saidi, 2011), generalized Fourier series method (Li and Daniels, 2002), characteristic orthogonal polynomials (Lal and Kumar, 2012), spline finite strip method (Tham and Szeto, 1990), boundary knot method (Shi et al., 2009), local Kriging meshless method (Zhang et al., 2014), Strong formulation finite element method (Fantuzzi and Tornabene, 2014a, 2014b) etc., which have been used to present the vibrational behaviour of plates of various geometries. Apart from these methods, the technique of differential transform method (DTM) developed by Zhou (1986) and Chen and Ho (1999) for solving the initial value problems has been employed by Malik and Huy Dang (1998) in studying the free vibration of continuous systems, particularly for thin beams. Later on, this method has been successfully extended to study the free axisymmetric vibrations of isotropic/FGM circular plates of uniform/non-uniform thickness with various boundary conditions/constraints in Refs. Yalcin et al. (2009) and Shariyat and Alipour (2010, 2011). Basically, DTM is a semi-analytical method based on Taylor series expansion. It is comparatively convenient to program than other classical methods which reduces the size of computations and gives highly accurate results.

Keeping in view that almost no work is available in the literature dealing with the dynamic buckling of FGM circular plates arising due to in-plane stressing, the differential transformation method has been employed to fill this gap by analysing the effect of in-plane force on the axisymmetric vibrations of FGM circular plates. According to this method, the governing differential equation of the motion of the present model gets reduce to a recurrence relation. Use of this recurrence relation in the boundary conditions together with the regularity condition, one obtains a set of two algebraic equations. These resulting equations have been solved using MATLAB to get the frequencies. The material properties i.e. Young's modulus and density are assumed to be graded in the thickness direction and these properties vary according to a power-law in terms of volume fractions of the constituents (Shamekhi, 2013). The natural frequencies are obtained for clamped and simply supported boundary conditions with different values of volume fraction index and in-plane force parameter. Mode shapes for the first three modes of vibration have been presented for the specified plates. A comparison of results has been given.

2. Mathematical formulation

Consider a FGM circular plate of radius a, thickness h, density ρ and subjected to uniform in-plane tensile force N_0. Let the plate be referred to a cylindrical polar coordinate system(R, θ, z), $z = 0$ being the middle plane of the plate. The top and bottom surfaces are $z = +h/2$ and $z = -h/2$, respectively. The line $R = 0$ is the axis of the