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Strength of a matrix with elliptic criterion reinforced by rigid inclusions with imperfect interfaces

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ABSTRACT

Elliptic effective strength criteria in the mean-deviatoric stress plane are encountered in porous media for a granular material made of rigid grains with cohesive frictional interfaces or a material with pores in a Drucker–Prager matrix. The macroscopic strength criterion of a heterogeneous material comprising a matrix with elliptic strength criterion reinforced by rigid inclusions with perfect or imperfect interfaces is studied. Considered imperfect interfaces follow either a Tresca or a Mohr–Coulomb strength criterion. Derived macroscopic criteria are shown to be a combination of a larger ellipse, which corresponds to the criterion for perfectly bounded interfaces, conditionally truncated by a smaller ellipse resulting from the activation of interfacial mechanisms. The activation of the interfacial mechanisms depends on the matrix and interfaces strength properties, inclusions concentration, as well as the macroscopic strain triaxiality ratio.

1. Introduction

Imperfect interfaces between constituents of heterogeneous media are increasingly understood to play a major role on the effective strength properties. Such imperfect interfaces can be characterized by a criterion the stress vector acting on the interface must not exceed. Thanks to homogenization methods, the strength of granular geomaterials has been investigated, successively considering the cases of rigid grains interfaced by a Tresca criterion (Dormieux et al., 2007), a2 frictional criterion (Maalej et al., 2009) or a cohesive frictional criterion (He et al., 2013), as well as the competition between Tresca interfacial and Von Mises intragranular strength (Dormieux et al., 2010).

Additionally, several types of porous media have been recognized to be governed by an elliptic effective strength criterion in the mean-deviatoric stress plane. For example, a composite made of pores in a matrix with a Von Mises (Barthélémy, 2005) or Drucker–Prager (Barthélémy, 2005; Maghous et al., 2009) strength criterion has an elliptic macroscopic strength criterion. Above a critical porosity threshold, the granular material with cohesive frictional interfaces considered in He et al. (2013) also proves to follow a similar criterion. Elliptic strength criteria are thus of great interest in geomechanics, for instance to describe the clay matrix of a shale. However, it is worth noting that this class of elliptic strength criteria is obtained only by applying the so-called modified secant modulii approach (Suquet, 1995) to the homogenization of ductile porous materials, which is precisely the method we intend to adopt in the present work for a second homogenization step. A Gurson-type analysis (Gurson, 1977) of the strength of porous materials would have led to the use of a different criterion for the clay matrix, but will not be considered here.

Driven by the thought-of example of shales for which the clay matrix is reinforced by silica or calcite inclusions (Fig. 1), the macroscopic strength of a material made of a matrix with an elliptic strength criterion reinforced by rigid inclusions is yet to be investigated. Furthermore, the degradation of this reinforcement, coming from matrix–inclusion interface imperfections, is of critical interest. The case of perfect interfaces will be compared to Tresca or Mohr–Coulomb interfaces. The present work addresses the imperfection of the interfaces under the hypothesis of ductility, whereas matrix–inclusion debonding had previously been studied in the context of fracture mechanics (see e.g. Mantič (2009) and Greco et al. (2013)).

This paper aims at addressing this strength issue using continuum micromechanics. From a technical point of view, Barthélémy

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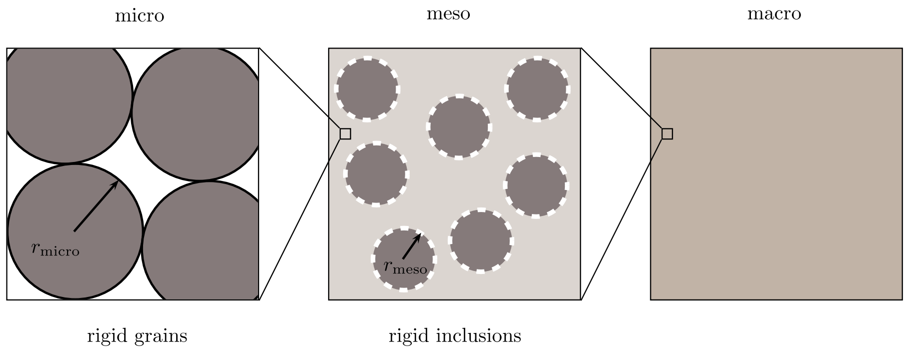

Fig. 1. Example of thought-of material. Cohesive frictional granular material reinforced by rigid particles with imperfect interfaces. According to the separation of scale principle, $r_{\text{micro}} \ll r_{\text{meso}}$. This paper deals with upscaling from meso to macro scales.

and Dormieux (2004) developed a strength homogenization method in which the macroscopic stress states lying on the boundary of the effective strength criterion domain are obtained by solving a fictitious non linear viscous problem. In turn, the non linear problem can be solved using secant or affine methods (Suquet, 1995, 1997) which rely on the solution to the associated linear problem. These methods proved successful to predict the strength of heterogeneous material, even in the presence of interface effects (Barthélémy, 2005; Barthélémy and Dormieux, 2004; Dormieux et al., 2010, 2006. 2007; He et al., 2013; Maalej et al., 2009; Maghous et al., 2009; Sanahuja and Dormieux, 2005).

To start with, the homogenization method is briefly recalled and the fictitious non linear problem is derived from the strength properties of the components in Section 2.

Next, the non linear homogenization method is presented in Section 3 after resolution of the linear problem underlying the fictitious problem arising from the previous section.

Finally, the macroscopic strength criteria are derived in Section 4 in the case of perfect, Tresca or Mohr–Coulomb interfaces.

Notations The second and fourth order identity tensors are respectively denoted by $\mathbf{1}$ and $\mathbb{I}$. The volumic and deviatoric projection tensors $\mathbb{J}$ and $\mathbb{K}$ are defined as $\mathbb{J} = \frac{1}{3}1\otimes 1$ and $\mathbb{K} = \mathbb{I} - \mathbb{J}$.

2. Limit state equations

2.1. Methodology

The aim of this article is to determine the effective strength of a composite made of a matrix reinforced by rigid inclusions with imperfect matrix–inclusion interfaces. A representative elementary volume (*rev*) Ω of this composite is introduced. It comprises a matrix (phase Ω_m) and rigid inclusions (phase Ω_i) with volume fraction ρ. Locally, the unit normal to the interface directed outwards from the inclusion is noted $\mathbf{n}$. The volume averages of a function a over the *rev* Ω, the matrix phase Ω_m, the inclusionary phase Ω_i, and all the interfaces Γ are respectively denoted $\overline{a}^\Omega = \overline{a}$, $\overline{a}^m$, $\overline{a}^i$ and $\overline{a}^\Gamma$.

A macroscopic stress state Σ applied to Ω is said to be admissible provided that there exists some microscopic stress field $\sigma(\mathbf{z})$ defined on the rev Ω which meets the following conditions (de Buhan, 1986; Suquet, 1983):

$$\begin{array}{ll} \mathbf{div}\boldsymbol{\sigma} = 0 & (\forall \mathbf{z}\in\Omega) \\ \boldsymbol{\sigma}(\mathbf{z})\in G(\mathbf{z}) & (\forall \mathbf{z}\in\Omega) \\ \overline{\boldsymbol{\sigma}} = \Sigma & \end{array} \tag{1}$$

where $G(\mathbf{z})$ denotes the domain of admissible microscopic stress states at point $\mathbf{z}$ in the rev Ω. The set of admissible macroscopic stress states is denoted by G^{hom}.

The strength properties of the constituents at the microscopic scale therefore need to be characterized. The domain G_m of admissible stress states in the matrix is defined by a strength criterion $f_m(\boldsymbol{\sigma})$ such that:

$$\boldsymbol{\sigma}\in G_m \Leftrightarrow f_m(\boldsymbol{\sigma})\leqslant 0.$$

In turn, the inclusions are supposed infinitely resistant. The emphasis of this paper is put on the limited strength of the matrix–inclusions interfaces: The inclusions may be not perfectly bounded to the matrix; instead, the strength of the interface is described by a criterion on the stress vector $\mathbf{T}$ acting on the matrix–inclusion interface:

$$\mathbf{T}\in G_\Gamma \Leftrightarrow f_\Gamma(\mathbf{T})\leqslant 0.$$

Γ denotes the set of matrix–inclusion interfaces and G_Γ defined above is the set of admissible stress vectors.

Equivalently, the domains G_m and G_Γ may be characterized by their support functions (Dormieux et al., 2006; Salencon, 1990). This is the so-called dual formulation. The support function $\pi_m(\mathbf{d})$ of the matrix strength is defined as

$$\pi_m(\mathbf{d}) = \sup\{\boldsymbol{\sigma} : \mathbf{d}, f_m(\boldsymbol{\sigma})\leqslant 0\},$$

where $\mathbf{d}$ and $\pi_m(\mathbf{d})$ physically represent a virtual strain rate and the associated dissipation. Likewise, the support function $\pi_\Gamma([\![\mathbf{v}]\!])$ of the interface criterion is

$$\pi_\Gamma([\![\mathbf{v}]\!]) = \sup\{\mathbf{T} : [\![\mathbf{v}]\!], f_\Gamma(\mathbf{T})\leqslant 0\},$$

where $[\![\mathbf{v}]\!]$ and $\pi_\Gamma([\![\mathbf{v}]\!])$ physically represent a virtual velocity jump across the interface and the associated dissipation.

The direct use of the definition (1) for the determination of G^{hom} is uneasy. Alternatively, as shown in Leblond et al. (1994),Barthélémy and Dormieux (2004), the boundary ∂G^{hom} of the macroscopic strength criterion can be retrieved by solving the following boundary value problem defined on the *rev* Ω

$$\begin{array}{ll} \mathbf{div}\boldsymbol{\sigma} = 0 & (\Omega) \\ \boldsymbol{\sigma} = \dfrac{\partial \pi_m}{\partial \mathbf{d}} & (\Omega_m) \\ \mathbf{T} = \dfrac{\partial \pi_\Gamma}{\partial [\![\mathbf{v}]\!]} & (\Gamma) \\ \boldsymbol{\sigma} = \mathbb{C}_i : \mathbf{d} & (\Omega_i) \text{ with } \mathbb{C}_i \to \infty \\ \mathbf{v}(\mathbf{z}) = \mathbf{D}\cdot\mathbf{z} & (\partial\Omega) \\ \mathbf{d} = \mathbf{grad}^s\mathbf{v} & (\Omega) \end{array} \tag{2}$$

This boundary value problem may be interpreted as a fictitious