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# Nonlinear stability analysis of thin FGM annular spherical shells on elastic foundations under external pressure and thermal loads

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ABSTRACT

To increase the thermal resistance of various structural components in high-temperature environments, the present research deals with nonlinear stability analysis of thin annular spherical shells made of functionally graded materials (FGM) on elastic foundations under external pressure and temperature. Material properties are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of constituents. Classical thin shell theory in terms of the shell deflection and the stress function is used to determine the buckling loads and nonlinear response of the FGM annular spherical shells. Galerkin method is applied to obtain closed – form of load – deflection paths. An analysis is carried out to show the effects of material, geometrical properties, elastic foundations and combination of external pressure and temperature on the nonlinear stability of the annular spherical shells.



## 1. Introduction

Nowadays, with the development of aesthetics, architectures and designs are becoming diversified and abundant. Thus, it requires study of shape and material of structures to be cared.

A considerable number of published researches in recent years have focused on the thermo-elastic, dynamic and buckling analyses of functionally graded material (FGM). This is mainly due to the increasing use of FGM as the components of structures in the advanced engineering. FGM consisting of metal and ceramic constituents have received remarkable attention in structural applications. Smooth and continuous change in material properties enable FGM to avoid interface problems and unexpected thermal stress concentrations. By high performance heat resistance capacity, FGM is now chosen to use as structural components exposed to severe temperature conditions such as aircraft, aerospace structures, nuclear plants and other engineering applications.

As a result, the problems relating to the thermo-elastic, dynamic and buckling analyses of plates and shells made of FGMs have attracted attention of many researchers, especially the FGM spherical shells. Shahsiah et al. (2006) extended their previous works for isotropic material to analyze linear stability of FGM shallow spherical shells subjected to three types of thermal loading. Ganapathi (2007) studied the problem, which is performed on the point of view of small deflection and the existence of type-bifurcation buckling of thermally loaded spherical shells. The nonlinear axisymmetric dynamic stability of clamped FGM shallow spherical shells has been analyzed by Prakash et al. (2007) using the first order shear deformation theory and finite element method. Bich and Tung (2011) have studied the nonlinear axisymmetric response of functionally graded shallow spherical shells under uniform external pressure including temperature effects. Huang (1964) reported an investigation on unsymmetrical buckling of thin isotropic shallow spherical shells under external pressure. Tillman (1970) investigated the buckling behavior of clamped shallow spherical caps under a uniform pressure load. Uemura (1971) employed a two term approximation of deflection to treat axisymmetrical snap buckling of a clamped imperfect isotropic shallow spherical shell subjected to uniform external pressure. Nonlinear static and dynamic responses of spherical shells with simply supported and clamped immovable edge have been analyzed by Nath and Alwar (1978) by making use of Chebyshev's series expansion. Nonlinear free vibration response, static response under uniformly distributed load, and the maximum transient response under uniformly distributed step load of orthotropic thin spherical caps on elastic foundation have been obtained by Dumir (1985). Buckling and postbuckling behaviors of laminated spherical caps subjected to uniform external pressure also have been analyzed by Xu (1991) and Muc (1992). Duc et al. (2014) investigated nonlinear axisymmetric response of FGM shallow spherical shells on elastic foundations.

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The annular spherical shell is one of the special shapes of the spherical shells. Despite the evident importance in practical applications, it is a fact from the open literature that investigations on the thermo-elastic, dynamic and buckling analyses of FGM annular spherical shell are comparatively scarce. There has been recently a few of publications on the annular shells. The most difficult in annular shell problems is complex calculations.

Alwar and Narasimhan (1992) investigated the axisymmetric nonlinear analysis of laminated orthotropic annular spherical shells, the object of this investigation is to give analytical solutions of large axisymmetric deformation of laminated orthotropic spherical shells including asymmetric laminates. Wu and Tsai (2004) studied the asymptotic DQ solutions of functionally graded annular spherical shells by combining the method of differential quadrature (DQ) with the asymptotic expansion approach. Dumir et al. (2005) analyzed axisymmetric dynamic buckling analysis of laminated moderately thick shallow annular spherical cap under central ring load and uniformly distributed transverse load, applied statically or dynamically as a step function load. Kiani and Eslami (2013) studied an exact solution for thermal buckling of annular FGM plates on an elastic medium, Bagri and Eslami (2008) generalized coupled thermo-elasticity of functionally graded annular disk considering the Lord–Shulman theory.

To the best of our knowledge, there has been recently no publication on solution of the nonlinear stability analysis (buckling and post-buckling) of thin FGM annular spherical shells on elastic foundations under temperature.

In this study, by using the classical thin shell theory, an approximate solution, which was proposed by Agamirov (1990) and was used by Sofiyev (2010) for truncated conical shells, the authors tried to give analytical solutions to the problem of nonlinear stability analysis of FGM thin annular spherical shells on elastic foundations under uniform external pressure and temperature.

## 2. Governing equations

Consider an annular spherical shell made of FGM resting on elastic foundations with radius of curvature $R$, base radii $r_1$,$r_0$ and thickness $h$. The FGM annular spherical shell is subjected to external pressure $q$ uniformly distributed on the outer surface as shown in Fig. 1.

The annular spherical shell is made from a mixture of ceramics and metals, and is defined in coordinate system ($\varphi$,$\theta$,$z$), where $\varphi$ and $\theta$ are in the meridional and circumferential direction of the shells, respectively and $z$ is perpendicular to the middle surface positive inwards.

Suppose that the material composition of the shell varies smoothly along the thickness by a simple power law in terms of the volume fractions of the constituents as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^k, \; -\frac{h}{2} \le z \le \frac{h}{2}, \qquad V_m(z) = 1 - V_c(z). \tag{1}$$

where $k$ (volume fraction index) is a non-negative number that defines the material distribution, subscripts $m$ and $c$ represent the metal and ceramic constituents, respectively.

The effective properties of FGM shallow spherical shell such as modulus of elasticity, the coefficient of thermal expansion, the coefficient of thermal conduction of FGM annular spherical shell can be defined as

$$[E(z), \alpha(z), K(z)] = [E_m, \alpha_m, K_m] + [E_{cm}, \alpha_{cm}, K_{cm}]\left(\frac{2z+h}{2h}\right)^k, \quad -\frac{h}{2} \le z \le \frac{h}{2}. \tag{2}$$

The Poisson ratio $\nu$ is assumed to be constant $\nu(z) = const$ and $E_{cm} = E_c - E_m$, $\alpha_{cm} = \alpha_c - \alpha_m$, $K_{cm} = K_c - K_m$.

The reaction–deflection relation of Pasternak foundation is given by Dumir (1985) amd Duc et al. (2014).

$$q_e = k_1 w - k_2 \Delta w$$

where $\Delta w = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}$ is a Laplace's operator, $w$ is the deflection of the annular spherical shell, $k_1$ is Winkler foundation modulus and $k_2$ is the shear layer foundation stiffness of Pasternak model.

In the present study, the classical shell theory is used to obtain the equilibrium and compatibility equations as well as expressions of buckling loads and nonlinear load-deflection curves of thin FGM annular spherical shells. For a thin annular spherical shell it is convenient to introduce a variable $r$, referred as the radius of parallel circle with the base of shell and defined by $r = R\sin\varphi$. Moreover, due to shallowness of the shell it is approximately assumed that $\cos\varphi = 1$, $Rd\varphi = dr$.

According to the classical shell theory, the strains at the middle surface and the change of curvatures and twist are related to the displacement components $u$,$v$,$w$ in the $\varphi$,$\theta$,$z$ coordinate directions, respectively, taking into account Von Karman–Donnell nonlinear terms as (Bich and Tung, 2011; Dumir, 1985; Xu, 1991; Duc et al., 2014).

$$\begin{aligned}
\varepsilon_r^0 &= \frac{\partial u}{\partial r} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2, & \chi_r &= \frac{\partial^2 w}{\partial r^2},\\
\varepsilon_\theta^0 &= \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{R} - \frac{w}{R} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2, & \chi_\theta &= \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2},\\
\gamma_r^\theta &= \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r} + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial \theta}, & \chi_{r\theta} &= \frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}.
\end{aligned} \tag{3}$$

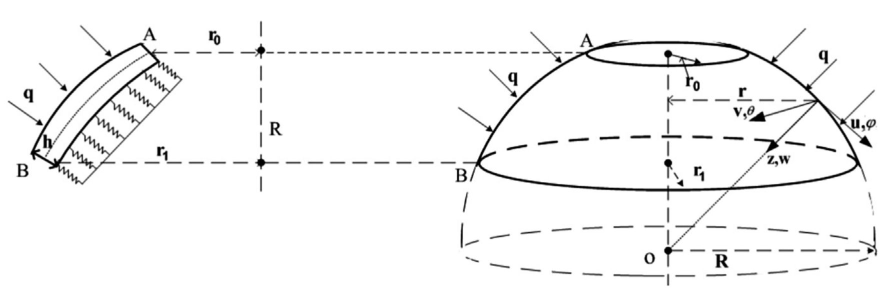


**Fig. 1.** Configuration of a FGM annular spherical shell.