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Continuous and discrete models for masonry like material: A critical comparative study

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ABSTRACT

The aim of this paper is to present a critical comparative review of different models that may be adopted for modelling the mechanical behaviour of masonry, with particular attention to microstructured models.

Several continuous and discrete models are discussed. Such models are based on the following assumptions: i) the structure is composed of rigid blocks; ii) the mortar is modelled as an elastic material or an elastic interface. The rigid block hypothesis is particularly suitable for historical masonry, in which stone blocks may be assumed as rigid bodies. For this type of masonry, mortar thickness is negligible if compared with block size, hence it can be modelled as an interface.

Masonry-like materials may be modelled taking into account their heterogeneity by adopting a heterogeneous Finite Element Model (FEM) or a Discrete Element Model (DEM). The former seems to be more representative of masonry, but it is computationally onerous and results interpretation may be difficult; the latter is limited to rigid block assumption and mortar joints modelled as interfaces. For this reason, continuous equivalent models may be suitable to investigate masonry behaviour. Continuum equivalent models provide, in an analytical form, constitutive functions, but Cauchy model may be not suitable to describe masonry behaviour due to not negligible size of heterogeneity (block size) with respect to masonry panel size. For this reason, micropolar equivalent continuum may be adopted.

By reference to the existing literature, a simple and effective DEM is adopted, in which masonry is modelled as a 'skeleton' having a behaviour depending on forces and moments transferred between blocks through the interfaces (mortar joints). Moreover for the micropolar equivalent continuum, an ad hoc enriched homogenised FEM is formulated by means of triangular elements. The proposed numerical models represent two possible simple approaches for solving heterogeneous problems. Such models are developed both by means of fast numerical routines and do not require specific computer codes, whereas the heterogeneous FEM may be studied by adopting a traditional FE code.

DEM and heterogeneous FEM are adopted to verify reliability and application field of Cauchy and micropolar continua. Moreover, sensitivity of micropolar model to the Representative Elementary Volume (REV) chosen is discussed. For these purposes, ad hoc FE models are adopted, with constitutive functions obtained from an identification procedure (both for Cauchy and micropolar continua). An extensive comparison between DEM, heterogeneous FEM and equivalent homogenous FEM is presented in some meaningful cases, taking into account also the effect of heterogeneity size on models behaviour.

1. Introduction

Masonry is a structural material obtained by composition of blocks connected or not by mortar joints. Particularity of this heterogeneous material is the heterogeneity size (size of block), that may be not negligible with respect to global size of structural element as in several composite materials. For this reason, in the last twenty years, several researchers developed models for studying masonry-like material adopting different approaches.

With this aim a heterogeneous FE model may be the more appropriate procedure to investigate this material type. Stafford Smith and Rahman (1972) were the first to adopt a rough heterogeneous FE model for determining stresses in brickwork walls;

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then, Page (Page, 1978; Ali and Page, 1988) adopted such type of model for fitting experimental results and taking into account the non-linear behaviour of mortar joints. However the limit of this approach lies in the difficulty to analyse macro-scale problems. As expected, the computational effort may be difficult to manage and the interpretation of numerical results may be not easy.

Then, a discrete model (DEM), based on the assumptions of rigid block behaviour and mortar joint modelled as interfaces, may be suitable for investigating masonry behaviour due to the small number of degrees of freedom (DOFs) involved in the analysis of masonry panels. These assumptions seem to be appropriate for historical masonry, in which stone block stiffness is very large if compared with mortar stiffness, allowing to assume blocks as rigid bodies, and mortar joint thickness is negligible if compared with block size, allowing to model mortar joints as interfaces. However, the assumption of rigid block imposes that boundary conditions must be referred only to block centres. This aspect may not be representative of actual mechanical behaviour. In the proposed DEM, masonry is seen as a 'skeleton' in which the interactions between rigid blocks are represented by forces and moments that depend on their relative displacements and rotations. Such model was adopted in the past by many authors for studying masonry behaviour in linear and non-linear fields (Masiani et al., 1995; Formica et al., 2002; Casolo, 2004, 2006). In particular, Cecchi and Sab (2004, 2009) defined a simple and effective DEM for studying the three-dimensional behaviour of masonry panels and for modelling random brickwork. Recently such model has been extended to the viscoelastic field by Baraldi and Cecchi (2014).

Discrete or distinct element models are widely adopted in other scientific fields such as rock mechanics (see for example the pioneering works of Cundall and Strack, 1979; Cundall, 1988). Limits in DEM approaches lie in the assumptions mentioned above, hence during the last decades the original model has been modified for taking into account the deformability of elements by introducing additional parameters or by introducing FE discretisations (Itasca, 1989). Some examples of evolution of the DEM are represented by commercial or open source codes (Itasca, 2000; Munjiza, 2004; Mahabadi et al., 2012) that are characterised by a larger computational effort with respect to the original DEM. Recently, a comparison between such models and a simple DEM has been carried on for studying masonry linear behaviour (Baraldi et al., 2013). Moreover, an exhaustive description of discrete models and their improvement up to recent years may be found in the work of Lemos (2007).

Although the DEM requires a small computational effort with respect to the heterogeneous FE model at micro-scale level and panel size level, it may be still unsuitable for studying masonry behaviour at macro-scale level. For the above mentioned reasons, continuous material equivalent to masonry were proposed. Among continuous models, homogenisation-identification procedures represent a consistent part of research. Indeed, homogenisation procedures allow to take into account different mechanical assumptions for blocks and mortar. Standard Cauchy continuous models are obtained applying periodic homogenisation techniques and considering the elastic behaviour of both brick and mortar (Anthoine, 1995; Cecchi and Sab, 2002; 2004). In the non-linear field some models exist in which blocks are assumed to be elastic and mortar is modelled with a coupled damage-friction behaviour (Milani et al., 2006; Sacco, 2009). In the non-linear field, both block and mortar may also display a non-linear behaviour (De Buhan and De Felice, 1997; Gambarotta and Lagomarsino, 1997; Pegon and Anthoine, 1997; Luciano and Sacco, 1998; Formica et al., 2002; Massart et al., 2007; Wei and Hao, 2009).

On the other hand, micropolar or higher order continua have also been adopted for masonry study. For micropolar continuum see for example Masiani et al. (1995), Masiani and Trovalusci (1996), Boutin (1996), Sulem and Mühlhaus (1997), Smyshlyaev and Cherednichenko (2000), Forest et al. (2001), Casolo (2009), Salerno and De Felice (2009), Addessi et al. (2010), De Bellis and Addessi (2011) and Pau and Trovalusci (2012). For higher order continuum see for example Stefanou et al. (2010), Bacigalupo and Gambarotta (2012) and Trovalusci and Pau (2014).

A crucial problem with the choice of homogenisation-identification procedures is not only how kinematic, dynamic, and constitutive prescriptions of a discrete system are transferred to the continuous one, but also which continuum may be more appropriate. Hence, constitutive functions of the continuous system may be different (Lofti and Benson Shing 1994, Lourenço and Rots, 1997; Del Piero, 2009).

The aim of this paper is to present a deep investigation of different models that may be adopted for modelling the mechanical behaviour of masonry, with particular attention to micro-structured models. At microscopic level, blocks are assumed to be rigid and mortar joints are modelled as elastic interfaces; a Cauchy standard continuum and a micropolar model – based on two different REVs – are considered. Then, an identification between the block structure and a plane continuum model is carried out by equating the mechanical work in the two models for a class of regular motions. Due to the hypotheses of the discrete model, the identification procedure turns out to be simpler than a homogenisation procedure and leads to the same results if blocks are assumed to be rigid. Hence, the constitutive function of the two-dimensional (2D) model is obtained from actual geometry and constitutive function of the discrete model. Such compatible identification procedure is adopted for all the models, in order to obtain equivalent continuous macroscopic constitutive functions, that turn out to be orthotropic starting from isotropic constitutive behaviour of block and mortar due to the arrangement of masonry texture.

At panel size level this paper presents a comparison between DEM and FEM in which constitutive continuum – Cauchy and micropolar – functions are obtained from an identification procedure. Furthermore, a FE heterogeneous model is taken into account, where constitutive functions of mortar and block are isotropic and where Young modulus of block is 10^4 time larger than Young modulus of mortar such as to simulate rigid block assumption. For representing the micropolar continuum and performing examples that can not be solved in analytical form, an enriched FE model is adopted, with triangular elements that take into account rotations as degrees of freedom. Recently in this field, several enriched FE models have been developed for studying the behaviour of generic micropolar elastic materials (Zhang et al., 2005, 2012; Beveridge et al., 2013). In particular, Providas and Kattis (2002) developed an enriched triangular FE model and proposed several patch tests.

The paper is organised as follows: in Section 2, a description of the 2D model is given and the mechanical power spent in its middle plane is defined. In Section 3, in a dual manner, the discrete model is described and the mechanical power, expanded to a generic couple of blocks at the interface, is defined. In Section 4a correspondence between a class of regular motions is defined for two portions of 2D and discrete models having the same size, and their mechanical power is equated. In this way, the stress measure in the plane is described as a function of the stress measure both for standard Cauchy model and for micropolar model. A constitutive linear isotropic elastic function for the mortar interface is adopted. Consequently, the above mentioned compatible identification leads to a constitutive orthotropic function. This procedure is applied to the case of a masonry panel with a running bond pattern and in Section 5 explicit formulas for this case are defined with reference to two different REVs. It must be noted that this methodological identification approach is embedded in linearised elasticity but may be extended to the non-linear case.