European Journal of Mechanics A/Solids 50 (2015) 59–69

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Analytic study of the onset of plastic necking instabilities during biaxial tension tests on metallic plates

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ARTICLE INFO

Article history:
Received 28 February 2014
Accepted 9 October 2014
Available online 18 October 2014

Keywords:
Plastic necking instabilities
Biaxial tension tests
Linear stability analysis

ABSTRACT

In this article, we study the onset of the development of plastic necking instabilities during dynamic tension tests on metallic plates biaxially loaded in their plane. This model applies whatever the thickness. The material is supposed to be homogeneous, isotropic, incompressible, elastoviscoplastic, to satisfy Von Mises' plasticity criterion and normality flow rule (damage and heat conduction are neglected). As Dudzinski and Molinari did in 1988 for static tests on very thin plates in the framework of generalized plane stress theory (thickness is supposed to vary slowly and slightly along loading directions) (Instabilité de la déformation viscoplastique en chargement biaxial, 1988, *Compte Rendu à l'Académie des Sciences de Paris* **307**, série 2, pages 1315–1321), we carry out a linear stability analysis. The flow is viewed as the sum of the mean homogeneous flow of the perfect plate, and of small perturbations $\delta\overrightarrow{v}$ of the velocity field, periodic along the x_1- and x_2- loading directions, growing exponentially. We search for the most unstable pair of wavelengths $(\lambda_1^{(d)}, \lambda_2^{(d)})$ and for the associated growth-rate $\theta^{(d)}$ (the dominant mode). Plastic deformation concentrates preferably along zero rate extension lines for non positive velocity gradient ratio $\alpha = D_{22}/D_{11}$ (**D** denoting deformation rate tensor), and along lines parallel to minor principal stress direction for biaxial stretching ($\alpha > 0$). For sufficiently viscous materials, inertia plays a negligible role (maximum plastic strain-rate considered in this paper equals $20\,\mathrm{s}^{-1}$, and thickness does not exceed 2 cm), the wavelength associated with the dominant mode is much greater than thickness, and Dudzinski and Molinari's model gives the associated growth-rate very accurately. This growth-rate is a root of a polynomial equation, that we re-establish starting from the equations of our 3*D* model. For non viscous materials, inertia is no longer negligible for non positive values of α.

1. Introduction

In the present article, we are interested in the onset of plastic necking instabilities during dynamic tension tests on metallic plates biaxially loaded in their plane, with constant velocities $\pm V_{01}\overrightarrow{e_1}$ and $\pm V_{02}\overrightarrow{e_2}$ applied at their edges (see Fig. 1). We carry out a 3*D* linear stability analysis (it applies whatever the thickness), and calculate the growth-rate of small perturbations $\delta\overrightarrow{v}$ of the velocity field of the mean homogeneous flow of the perfect plate, symmetrical with respect to the median plane (and therefore representative of local thinnings), periodic along the x_1- and x_2- loading directions, growing exponentially (see Fig. 2). We get the dominant mode – *i.e.* the most unstable pair of wavelengths (λ_1, λ_2) – and therefore the plastic deformation localization zones.

The material is supposed to be homogeneous, isotropic, incompressible, elastoviscoplastic, to obey Von Mises (1913) plasticity criterion and normality flow rule. Its yield strength Y depends on plastic strain ε_p, plastic strain-rate $\dot{\varepsilon}_p$ and absolute temperature T_K. Its shear modulus G depends on T_K. Damage, thermal expansion and heat conduction are neglected. The flow is supposed to be adiabatic, and plastic work is fully converted into heat.

In this framework, the main results are:

1. for non positive strain-rate ratio α $(-1 \le \alpha = D_{22}/D_{11} \le 0)$ (**D** denotes deformation rate tensor), plastic strain concentrates along zero rate extension lines, that are inclined at Hill's angle with respect to minor principal stress direction x_2 (Hill, 1952):

$$\psi_{\mathrm{Hill}} = \arctan(n_2/n_1) = \arctan\left(\sqrt{-D_{22}/D_{11}}\right) \qquad (1)$$

This localization is observed in numerous experiments (Nadái, 1950), for isotropic materials. For static tests on very thin plates, it was shown by Hill (1952), with a bifurcation analysis, for a non viscous rigid plastic material (G infinite) obeying Hollomon's

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http://dx.doi.org/10.1016/j.euromechsol.2014.10.002
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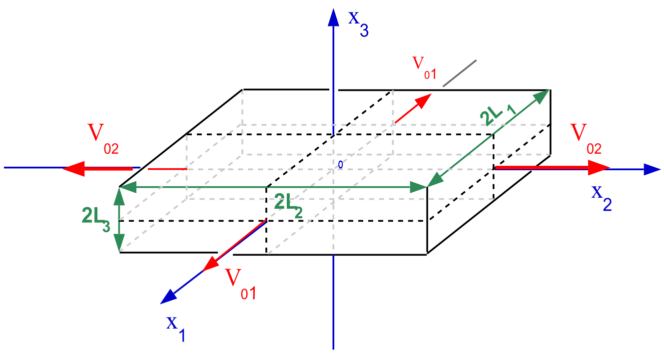

Fig. 1. Plate biaxially loaded in tension in its plane. We take x_1 as major principal (Cauchy) stress direction, and x_2 as minor principal stress direction: $0 \le |\Sigma_{22}| \le \Sigma_{11}$. The velocity gradient ratio $\alpha = D_{22}/D_{11}$ ($D_{11} > 0$) (**D** denoting strain-rate tensor) is between -1 (simple shear loading in the (x_1, x_2) plane) and 1 (balanced stretching). It equals $-1/2$ in uniaxial tension along x_1-direction, and 0 in plane tension in the (x_1, x_3) plane.

constitutive law ($Y \propto \varepsilon_p^n$, $n > 0$) (Hollomon, 1945), that, as elongation ε_1 reaches the value:

$$\varepsilon_1^* = \frac{n}{1 + D_{22}/D_{11}} \tag{2}$$

localization bands, whose orientation is given by Hill's angle, appear instantaneously. In 1988, carrying out a linear stability analysis, Dudzinski and Molinari (1988) retrieved plastic deformation localization along zero rate extension lines with these hypotheses. They showed that, when elongation ε_1 tends to ε_1^*, the dominant mode's angle tends to Hill's angle, and the related growth-rate tends to infinity.

Both analyses were carried out in the framework of generalized plane stress theory (Hill, 1950; Hodge, 1951); the plate is supposed to be very thin; thickness is supposed to vary slowly and slightly along the loading directions, and the space between localization lines is supposed to be much greater than thickness. Our 3*D* analysis retrieves the orientation of the localization lines following Hill's angle. Due to inertia, for non-viscous materials and as long as work-hardening prevails against thermal softening, localization occurs in a finite duration, and the dominant mode's wavelength is an intermediate one.

2. for positive strain-rate ratios ($0 < \alpha < 1$), plastic deformation concentrates along lines parallel to minor principal stress direction x_2 (the wavelength $\lambda_2^{(d)}$ associated with the dominant mode is infinite). This orientation has also been found by Dudzinski and Molinari (1988) for long wavelength perturbations. The growth-rate associated with the dominant mode is all the smaller (and the wavelength $\lambda_1^{(d)}$ all the greater) that α is greater. Hill's bifurcation analysis concludes that ductility is infinite during biaxial stretching tests, because it applies only to instantaneous processes.

Plane strain tension ($\alpha = 0$; the dimension of the plate along the x_2- loading direction is infinite) is a particular case of the 3*D*– model we present here. Numerous publications have been devoted to that case. Let us consider the ones written as we do in the framework of the so-called "J_2- Flow Theory" (Von Mises plasticity criterion and the associated normality flow rule are supposed to be satisfied). In 1977, Hutchinson et al. (1978) carried out a rigorous 2*D* Linear Stability Analysis to study the onset of necking plastic instabilities during static plane tension tests on rigid viscoplastic materials obeying Norton's law (Norton, 1929). Viscosity damps the development of short wavelength instabilities (in comparison with thickness); therefore the dominant mode lies in the field of long wavelengths. In 1994, keeping the same constitutive law, Fressengeas and Molinari (1994) showed that inertia stabilizes the longest wavelengths. Thus, the wavelength associated with the dominant mode is an intermediate one. In 2010, Mercier et al. (2010) generalized this model to any analytic constitutive law $Y(\varepsilon_p, \dot{\varepsilon}_p, T_K)$, and they retrieved the order of magnitude of the number of necks (and the time at which they appear) during the expansion of hemispherical metallic shells driven by explosive charge. Carrying out numerical simulations, they determined the plane strain loaded zone at different instants of time (this zone is the most unstable zone, as we shall see further), to which they applied their Linear Stability Analysis.

Applied to plane strain tension ($\alpha = 0$), the 3*D*– model we present here retrieves important published results (Jouve, 2014), in particular by Fressengeas and Molinari (1992, 1994), and also by Hill and Hutchinson (1975) and Young (1976). As other authors carrying out simpler approaches (see (Hart, 1967) for example), we retrieve the criterion of Considère (1885) in the absence of viscous effects; the first instabilities, that are long wavelength ones for ductile metals, appear when the applied force is maximum (see Jouve (2013), Subsection 3.5). Some important similarity laws have been established for non viscous materials. In particular, for a constitutive law in the form $Y(\varepsilon_p)$, a given deformation level ε_p and a given product $\gamma_1 L_3$ (with $\gamma_1 = 2\pi/\lambda_1$), the growth-rate θ is inversely proportional to the square root of mass density ρ: $\theta \propto 1/\sqrt{\rho}$ (see Jouve (2013), Subsection 3.4). In this article, we shall see that this similarity law is valid for non positive α, in the absence of viscous effects.

1.1. Outline

After briefly recalling the evolution equations of the mean homogeneous ground flow (Section 2), and the equations of the linear stability analysis (section 3 – see (Jouve, 2010) for details), we search for the dominant mode in Section 4, for different values of α and different constitutive laws. We assess the field of values of α such as inertia and elasticity play a negligible role, with dominant mode having long wavelength compared to thickness. In these cases, the model of Dudzinski and Molinari suffices to predict very accurately the development of necking. The growth-rate of these long wavelength instabilities is a root of a simple polynomial equation, that we retrieve in section 5 starting from the equations of our 3*D* linear stability analysis.

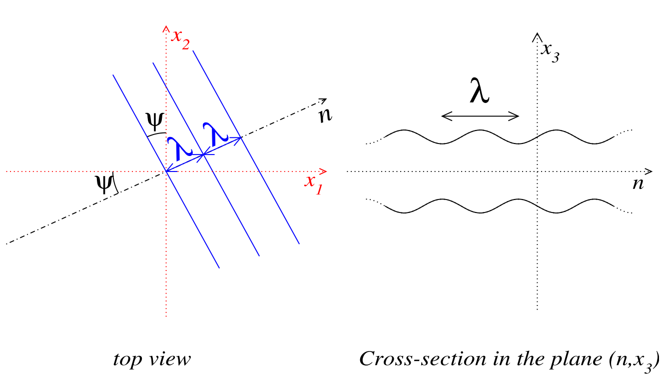

Fig. 2. Periodically spaced thinnings, due to a perturbation $\delta\vec{v}$ of the velocity field, symmetrical with respect to the median plane, written as: $\delta\vec{v} = e^{\theta t} e^{i2\pi n/\lambda} \vec{F}(x_3)$ (F_1 and F_2 are even functions of x_3, F_3 is an odd function of x_3).