Contents lists available at ScienceDirect

## European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

# Nonlinear free vibration analysis of variable stiffness symmetric skew laminates

### A. Houmat

Department of Mechanical Engineering, University of Tlemcen, B.P. 230, Tlemcen 13000, Algeria

#### A R T I C L E I N F O

Article history: Received 27 October 2013 Accepted 24 October 2014 Available online 4 November 2014

*Keywords:* Skew laminate Variable stiffness Nonlinear free vibration

#### ABSTRACT

A skew *p*-element is developed for the nonlinear free vibration of variable stiffness symmetric skew laminates. The governing equations are based on thin plate theory and Von Karman strains. The fundamental frequencies and normal modes are computed for fully clamped edge conditions. The equations of motion are derived using Lagrange's method. By employing the harmonic balance method, the transformation from time to frequency domain is facilitated. The nonlinear equations are solved using the iterative technique known as the linearized updated mode method. The numerical results are validated with the help of convergence tests and comparisons with published data. New results are presented for variable stiffness symmetric skew laminates with different fiber configurations showing the effects of variation in skew angle on frequency, normal mode, and degree of hardening.

© 2014 Elsevier Masson SAS. All rights reserved.

#### 1. Introduction

Skew laminates are commonly used in modern structural applications because of their high strength-to-weight ratio and excellent resistance to fatigue. To guarantee a suitable design, the vibration characteristics of those structures must be known. Generally, it is difficult to obtain exact solutions for the free vibration of skew laminates. Hence, one must make use of numerical methods to obtain approximate solutions. The numerical methods which have been proposed for the solution of the linear free vibration of constant stiffness skew laminates are the finite element method (Krishnan and Deshpanda, 1992; Krishna Reddy and Palaninathan, 1999), Ritz method (Kapania and Singhvi, 1992; Anlas and Goker, 2001; Han and Dickinson, 1997), Green's function method (Hosokawa et al., 1996), B-spline Ritz method (Wang, 1997a,b), and finite strip transition matrix method (Ashour, 2009). Nonlinearity arises when the laminate vibrates at large amplitudes. The nonlinear free vibration of constant stiffness skew laminates has been solved numerically using the finite element method (Singha and Ganapathi, 2004; Singha and Daripa, 2007) and differential quadrature method (Malekzadeh, 2007, 2008). The nonlinear free vibration of variable stiffness rectangular laminates has been solved numerically using a rectangular *p*-element based

http://dx.doi.org/10.1016/j.euromechsol.2014.10.008 0997-7538/© 2014 Elsevier Masson SAS. All rights reserved. on nonlinear first-order shear deformation plate theory (Ribeiro and Akhavan, 2012).

The literature review unveils that there are no solutions to the nonlinear free vibration of variable stiffness skew laminates. This paper aims to fill this gap by obtaining new results for the fundamental frequencies and normal modes of variable stiffness symmetric skew laminates using a skew *p*-element based on thin plate theory and von Karman strains. Recently, the author (Houmat, 2013) obtained new results for the fundamental frequencies and normal modes of variable stiffness rectangular laminates using a rectangular *p*-element based on thin plate theory and von Karman strains. The design space depended only on the length. This work is an extension of the author's previous study to variable stiffness skew laminates. Herein, the design space depends on the oblique width and skew angle. The principal goal of this study is to validate the numerical results with the aid of convergence tests and comparisons with published data. In addition, a parametric analysis is used to study the normal modes and degree of hardening by changing the skew angle.

#### 2. Characteristics of variable stiffness skew laminate

The slope of the basic fiber curve is assumed to vary linearly with x' from  $T_0$  at the middle to  $T_1$  at a distance  $b\cos\phi/2$  from the origin in which  $\phi$  denotes the skew angle and b denotes the oblique width. The linear variation of slope has the advantage that it yields closed-form expressions for the curve equation and curvature. A







E-mail address: ahoumat@hotmail.com.

basic fiber is symbolized by  $\langle T_0|T_1\rangle$ . Fig. 1 shows the basic fiber curve. The ordinate y' and slope  $\theta$  of the fiber curve are expressed as

$$y' = \begin{cases} \frac{b\cos\phi}{2(T_1 - T_0)} \left\{ -\ln[\cos T_0] + \ln\left[\cos\left(T_0 - \frac{2(T_1 - T_0)}{b\cos\phi}x'\right)\right] \right\} & \text{for} & -\frac{b}{2}\cos\phi \le x' \le 0\\ \frac{b\cos\phi}{2(T_1 - T_0)} \left\{ \ln[\cos T_0] - \ln\left[\cos\left(T_0 + \frac{2(T_1 - T_0)}{b\cos\phi}x'\right)\right] \right\} & \text{for} & 0 \le x' \le \frac{b}{2}\cos\phi \end{cases}$$
(1)

$$\theta = \frac{\pi}{2} + \arctan\left(\frac{dy'}{dx'}\right)$$

$$= \begin{cases} \frac{\pi}{2} + T_0 - \frac{2(T_1 - T_0)}{b\cos\phi}x' & \text{for } -\frac{b}{2}\cos\phi \le x' \le 0 \\ \frac{\pi}{2} + T_0 + \frac{2(T_1 - T_0)}{b\cos\phi}x' & \text{for } 0 \le x' \le \frac{b}{2}\cos\phi \end{cases}$$
(2)

in which x' = y.

The rest of fibers are conceived by relocating the basic fiber fixed distances in the direction parallel to the *y*'-axis. Fig. 2 shows a two-layer skew laminate ( $\phi = 15^{\circ}$ ). The skew laminate is symbolized by  $[\mp \langle 30^{\circ} | 60^{\circ} \rangle]$ . To prevent fiber kinking, the maximum curvature must not surpass 3.28 m<sup>-1</sup> (Waldhar, 1996).

The curvature  $\kappa$  is given by

$$\kappa = \frac{2(T_1 - T_0)}{b\cos\phi} \cos\left[T_0 + \frac{2(T_1 - T_0)}{b\cos\phi}x'\right] \text{ for } 0 \le x' \le \frac{b}{2}\cos\phi \qquad (3)$$

#### 3. Formulation

The relationships between dimensionless and Cartesian co-ordinates are

$$\xi = \frac{2}{a}(x - y \tan \phi) \tag{4}$$

$$\eta = \frac{2y}{b\cos\phi} \tag{5}$$



Fig. 1. Basic fiber curve and slope.

The displacements *u*, *v*, and *w* are written as

$$u = \sum_{k=1}^{p+1} \sum_{l=1}^{p+1} \overline{q}_{2l-1}(t) g_k(\xi) g_l(\eta)$$
(6)

$$\nu = \sum_{k=1}^{p+1} \sum_{l=1}^{p+1} \overline{q}_{2J}(t) g_k(\xi) g_l(\eta)$$
(7)

$$w = \sum_{k=1}^{p+1} \sum_{l=1}^{p+1} q_l(t) f_k(\xi) f_l(\eta)$$
(8)

in which *p* denotes the polynomial order, *t* denotes time, and J = l + (k - 1)(p + 1).

Expressions for the shape functions  $g_{\alpha}$  ( $\alpha = 3, 4 \dots 11$ ) and  $f_{\beta}$  ( $\beta = 5, 6 \dots 11$ ) can be found in Houmat (2012).

The relationships between strains and displacements are

$$\varepsilon_{XX} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \tag{9}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 \tag{10}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}$$
(11)

The relationships between curvatures and transverse displacement are



**Fig. 2.**  $[\mp \langle 30^{\circ} | 60^{\circ} \rangle]$ skew laminate with  $\phi = 15^{\circ}$ .

Download English Version:

## https://daneshyari.com/en/article/778002

Download Persian Version:

https://daneshyari.com/article/778002

Daneshyari.com