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Analytical modelling for vibration analysis of partially cracked orthotropic rectangular plates

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ABSTRACT

An analytical model is presented for vibration analysis of a thin orthotropic plate containing a partial crack located at the centre of the plate. The continuous line crack is parallel to one of the edges of the plate. The equation of motion of an orthotropic plate is derived using the equilibrium principle based on Classical Plate Theory. The crack terms are formulated using the Line Spring Model. Using Berger's formulation for the in-plane forces and Galerkin's method for solution, the derived equation involving a cubic nonlinear term is converted into the Duffing equation. The effect of nonlinearity is established by deriving the frequency response equation for the cracked plate using the method of multiple scales. The influence of crack length and boundary conditions on the fundamental frequency of square and rectangular plate is demonstrated for three boundary conditions. It is found that the vibration characteristics are affected by the presence of crack. Further, it is deduced that the presence of a crack across the fibres affects the fundamental frequency more as compared to a crack along the fibres. The effect of varying elasticity ratio on fundamental frequency of cracked plate is also established.

1. Introduction

Orthotropic plates find many applications in the areas of structural mechanics, shipbuilding, civil and mechanical installations. As a simplification, composite plates are often modelled as orthotropic. The orthotropic behaviour can be by the use of materials having orthotropic stress strain relationships. In case of isotropic plate, unequal stiffening in the two mutually perpendicular directions makes it orthotropic (Biancolini et al., 2005; Xing and Liu, 2009). In order to cater the wide applications of such plates, dynamic analysis, especially, knowledge of vibration characteristics becomes important. An exhaustive literature review on vibration of plates has been given by Leissa (1969) in his monograph. Rayleigh–Ritz method, one of the well known methods is widely used to provide approximate solutions to the natural frequencies of orthotropic plates and many researchers have effectively applied this method. The conventional Navier and Levy type solution methods (Xing and Liu, 2009; Szilard, 2004) can also be extended to orthotropic plates having either all edges simply supported or two opposite edges simply supported. Kshirsagar and Bhaskar (2008) applied superposition approach developed by Bhaskar and Sivaram (2008), for free vibration of orthotropic plates involving mixed boundary conditions. Xing and Liu (2009) obtained exact natural frequencies for thin orthotropic rectangular plates by the method of separation of variables. Biancolini et al. (2005) deduced an approximate way of finding fundamental modes of free vibration of thin orthotropic plates starting with a general approximate formula. Although the Classical Plate Theory over predicts the fundamental frequencies, literature shows that many researchers have employed it for orthotropic thin plates (Biancolini et al., 2005; Xing and Liu, 2009). The presence of a defect in the form of crack or hole affects the vibration characteristics. Rice and Levy (1972) represented a continuous line crack by distributed line spring with stretching and bending compliances thus formulating the Line Spring Model (LSM) using Classical Plate Theory. Gutierrez et al. (2000) considered circular and rectangular holes in their work on transverse vibration of isotropic and orthotropic plates. They concluded that, for a clamped plate the fundamental frequency increases with increase in hole diameter due to dynamic stiffening. Huang et al. (2011) presented results for fundamental frequencies, applying the Ritz method for an isotropic rectangular plate with a through crack. They proposed new admissible functions to consider the singularities at the crack tips and concluded that increase of crack length decreases the fundamental

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frequency. Extending their work, Huang et al. (2012) considered through internal cracks for three dimensional vibration analysis of functionally graded rectangular plates. Bachene et al. (2009) applied extended finite element method for the solution of cracked plates. Viola et al. (2013) applied differential quadrature finite element method to investigate dynamics of thick composite plates containing a through crack. Natarajan et al. (2011a) applied the extended finite element method proposed by Bachene et al. (2009) and considered 20 degrees of freedom for a 4 node quadrilateral plate element in their application for cracked functionally graded plates. The authors provided extensive results on crack length, orientation, crack location and for multiple cracks. Baiz et al. (2011) studied linear buckling of cracked isotropic plates by smoothed curvatures and extended finite element method. Recently Ismail and Cartmell (2012) presented an analytical approach for partially cracked isotropic plate, wherein the crack is inclined to the edges of the plate. Their approach is based on LSM and crack compliances are used to represent the surface crack. More recently, Bose and Mohanty (2013) considered arbitrary position and orientation of a part through crack in a thin isotropic plate for vibration analysis and deduced that the orientation of crack affects the vibration characteristics of the plate. Khadem and Rezaee (2000) presented vibration analysis of cracked rectangular plate considering bending compliance only and proposed new comparison functions. They deduced that presence of crack at a specific location affects the natural frequencies differently. Krawczuk et al. (2001) considered elastic–plastic through crack in their formulation by finite element scheme. Israr et al. (2009) developed an approximate analytical model for vibrations of a cracked isotropic plate using LSM to represent bending and tensile forces by net ligament at crack location. In their work, a surface crack located at the centre is parallel to one of the edges of the plate. The authors introduced bending and stretching force effects due to the crack. Their study is based on Classical Plate Theory and concluded that the fundamental frequency decreases as crack length increases. Recently, Huang and Chan (2014), in their application of Ritz method used moving least squares interpolation functions for vibration of cracked plate. Jha et al. (2013) applied higher order shear and normal deformation theory for free vibration analysis of functionally graded plates.

Vel and Batra (2002) presented an exact solution for three-dimensional thermo-elastic deformations of a simply supported functionally graded thick rectangular plate using the power series method. They reduced the partial differential equations governing the thermoelastic deformations to a set of ordinary differential equations by suitable displacement and temperature functions. The authors compared their results with classical, first order and third order shear deformation theories for functionally graded plates. Extending their work (Vel and Batra, 2004) to free and forced vibrations of functionally graded plates (FGP), the authors presented exact natural frequencies for FGP. They also considered the effect of varying microstructure in the thickness direction. Batra and Jinb (2005) combined the finite element method with the first order shear deformation theory to analyse free vibrations of functionally graded anisotropic plates. Ferreira et al. (2006) presented results for natural frequencies of FGP by a meshless method based on first and third order theories. They used the global collocation method with radial basis functions in their approach. Ferreira et al. (2009) extended the radial basis function approach to analyse natural frequencies of orthotropic, monoclinic and hexagonal material thick plates. The authors demonstrated the efficacy of the pseudospectral method for eigenvalue problems. Ahmad Akbari et al. (2010) applied the Meshless Local Petrov–Galerkin method to simulate the propagation of a thermoelastic wave through FGP and presented results for dynamic displacement and stress fields under transient thermal field. The work of Rahimabadi et al. (2013) shows the effect of cutouts and cracks on vibrations of FGP. The authors used an enriched shear flexible 4-noded quadrilateral element with a Heaviside function to affect the displacement across the discontinuity and asymptotic branch functions to accommodate the singularity around the crack tip. They presented results for natural frequencies of FPG affected by cutout size and crack lengths in the presence of thermal environment. The work of Natarajan et al. (2011b, 2013, 2014, 2012) shows the application of finite element method to study static bending, free vibration, mechanical and thermal buckling of intact and cracked FGP. The authors used 8-noded shear flexible element (Natarajan et al., 2011b), enriched 4-noded quadrilateral element (Natarajan et al., 2013), 3-noded triangular element (Natarajan et al., 2014) and NURBS basis functions based iso-geometric finite element method (Natarajan et al., 2012), based on the work of Valizadeh et al. (2013), Shojaee et al. (2012). Shear correction factors are evaluated using the energy equivalence principle. The plate kinematics are based on first order shear deformation theory and they concluded that the natural frequencies of FGP decrease as a result of increase in gradient index, through crack length and temperature gradients. The supersonic flutter behaviour of simply supported thin, cracked FGP submerged in supersonic flow is also studied by Natarajan et al. (2013) wherein, the crack is modelled independent of the underlying mesh. They concluded that the critical frequency and critical pressure are at a minimum when the crack is aligned to flow angle. A shear locking free, simple and efficient plate element formulation for FGP is presented by Yin et al. (2014) to study bending, buckling and free vibration. Erdogan and Wu (1996, 1997) presented mode *I* stress intensity factors for cracked FGP. They also considered the effect of thermal environment (Erdogan and Wu, 1996). Reddy (2000) presented deflection and stress analysis of FGP based on third order shear deformation theory with linear and nonlinear finite element models. Ganapathi et al. (2006) studied the critical buckling of simply supported functionally graded skew plates subjected to mechanical loads. Their finite element formulation is based on first order shear deformation theory and they presented results for critical buckling load as affected by gradient index, aspect ratio and skew angle. The literature (Huang et al., 2011; Ismail and Cartmell, 2012; Khadem and Rezaee, 2000; Krawczuk et al., 2001; Israr et al., 2009; Natarajan et al., 2011b, 2013, 2014, 2012; Wu and Law, 2004; Wu and Shih, 2005) shows that the natural frequencies of isotropic plate depend on several factors such as crack length, plate geometry, crack orientation and crack location. Thus, it is important to analyse the effect of partial crack on the fundamental frequencies of thin orthotropic plates. The present work addresses this by proposing an analytical model.

The present work references the methods and analytical model deduced by Israr et al. (2009) for an isotropic plate containing a surface crack, extends and applies it to a thin orthotropic plate. The line spring model is modified to accommodate the orthotropic nature of the plate. Results for fundamental frequencies are presented considering the influence of boundary conditions and crack length. The presence of a hole in a plate reduces both mass and stiffness, in the present work the crack is in the form of continuous line and is parallel to one of the edges of the plate. Such a line crack affects only the stiffness of the plate and not its mass. The configuration of the cracked plate for analysis is shown in Fig. 1. Fig. 1(a) shows a partial crack parallel to the x axis and Fig. 1(b) shows a crack parallel to the y axis. L_1 and L_2 are the plate dimensions along x and y axes respectively. The plate of thickness h has a partial crack of length $2a$ along the fibres and is parallel to x axis, whereas its