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Asymptotic analysis of cracks in the adhesive layer of an orthotropic sandwich structure

H.G. Beom*, C.B. Cui, H.S. Jang

Department of Mechanical Engineering, Inha University, 253 Yonghyundong, Incheon 402-751, Republic of Korea

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ABSTRACT

We analyze the asymptotic problems of an interface crack in an orthotropic sandwich structure and a semi-infinite subinterface crack in an orthotropic bimaterial. Stress intensity factors for the asymptotic problems are obtained from the path independence of the *J* integral; these involve only each undetermined dimensionless function. The dimensionless functions are shown to depend on three bimaterial parameters and two orthotropy parameters. By employing an orthotropy rescaling technique, the explicit dependence of one orthotropy parameter on the dimensionless functions is obtained. The effects of the other four material parameters on the dimensionless functions are numerically determined. The asymptotic problem of a crack in an adhesive layer of a sandwich structure is also considered. The stress intensity factors for the crack in an adhesive crack are obtained, with only one dimensionless function undetermined. An approximate formula for the dimensionless function is obtained in terms of the solutions of dimensionless functions for both the interface crack and the subinterface crack.

1. Introduction

Cracking in an adhesive layer of a sandwich structure is frequently observed. A crack propagates with various local cracking morphologies such as a straight crack, interface crack, alternating crack, and wavy crack. If the adhesive layer thickness is small compared to all other lengths of the sandwich structure, the influence of the adhesive layer on elastic fields distant from the crack tip is negligible. Thus, some asymptotic problems of cracks in an adhesive layer of a sandwich structure have been investigated. Suo and Hutchinson (1989) solved the asymptotic problem of a crack lying along one interface in an isotropic sandwich structure. They employed a dislocation density function technique to obtain the interface stress intensity factors. Fleck et al. (1991) analyzed the modified asymptotic problem of a crack in an isotropic adhesive layer to investigate possible crack propagation paths in the adhesive layer. In the modified asymptotic problem, remote fields at infinity are given by the singular fields and elastic *T* stress near the crack tip without an adhesive layer. Akisanya and Fleck (1992) studied an alternating crack in isotropic sandwich specimens. The stress intensity factors were evaluated using the finite element method, which was used to predict the wavelength of the alternating crack. Recently, experimental works on the determination of the interface fracture toughness of an adhesive joint (Shi et al., 2006) and the enhancement of adhesive joint strength (Sturiale et al., 2007) have been performed. However, previous works have been limited mainly to cases of isotropic sandwich structures.

Stress fields in an isotropic bimaterial with prescribed tractions over its whole boundary depend on two dimensionless bimaterial parameters known as Dundurs parameters. For a bimaterial consisting of dissimilar orthotropic material, six independent material parameters are needed to describe inplane stress fields (Beom and Atluri, 1995a; Ting, 1995). In general, stress fields in a sandwich structure in which an orthotropic adhesive layer joins two identical orthotropic substrates are dependent upon the six material parameters. Too many material parameters make it difficult to present in compact form the numerical results of stress intensity factors for the orthotropic sandwich problem.

The purpose of the present study is to analyze cracks in an adhesive layer of an orthotropic sandwich structure under plane deformation. We consider asymptotic problems of cracks on the interface, and in an adhesive layer of the sandwich structure. In the asymptotic problems, the remote fields at infinity are given by the fields near the crack tip without an adhesive layer. Invoking the path independence of the *J* integral (Rice, 1968), stress intensity factors for the asymptotic problems can be evaluated. Each expression of the stress intensity factors for the asymptotic

* Corresponding author. Tel.: +82 32 860 7310; fax: +82 32 868 1716.
E-mail address: hgbeom@inha.ac.kr (H.G. Beom).

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problems involves only one undetermined dimensionless function. In order to explore the dependence of the dimensionless functions on material constants, the Stroh formalism (Eshelby et al., 1953; Stroh, 1958) and an orthotropy rescaling technique (Suo, 1990a) are employed. A reduction in the number of material parameters affecting the stress intensity factors is made. The dimensionless function for the interface crack depends on five material parameters, whereas four material parameters affect the dimensionless function for the crack in an adhesive layer. We show that the dimensionless function for the interface crack can be obtained from the dimensionless function for a transformed problem that depends on four material parameters. The approximate formula of the dimensionless function for the crack in an adhesive layer is obtained in terms of solutions for the interface crack and sub-interface crack.

2. Interface crack in a sandwich structure

2.1. Formulation

Consider the asymptotic problem of the interface crack in a sandwich structure as shown in Fig. 1. The sandwich structure consists of two substrates with material 1, and an adhesive layer with material 2. Each material is assumed to be orthotropic. The principal material axes of the orthotropic materials are coincident with the Cartesian coordinate axes x_1 and x_2. The crack lies on the interface between the upper substrate and the adhesive layer along the negative x_1 axis. The adhesive layer has thickness h. The crack surface is assumed to be traction free, and the interfaces are bonded perfectly. The boundary conditions on the crack surfaces and the continuity conditions of displacements and tractions on the bonded interface are given by

$$\begin{aligned}
&\begin{Bmatrix} \sigma_{21}(x_1,0^{\pm}) \\ \sigma_{22}(x_1,0^{\pm}) \end{Bmatrix} = \mathbf{0}, \quad x_1<0,\\
&\begin{Bmatrix} u_1(x_1,0^{+}) \\ u_2(x_1,0^{+}) \end{Bmatrix} = \begin{Bmatrix} u_1(x_1,0^{-}) \\ u_2(x_1,0^{-}) \end{Bmatrix}, \quad x_1>0\\
&\begin{Bmatrix} \sigma_{21}(x_1,0^{+}) \\ \sigma_{22}(x_1,0^{+}) \end{Bmatrix} = \begin{Bmatrix} \sigma_{21}(x_1,0^{-}) \\ \sigma_{22}(x_1,0^{-}) \end{Bmatrix}, \quad x_1>0,\\
&\begin{Bmatrix} u_1(x_1,-h^{+}) \\ u_2(x_1,-h^{+}) \end{Bmatrix} = \begin{Bmatrix} u_1(x_1,-h^{-}) \\ u_2(x_1,-h^{-}) \end{Bmatrix}, \quad -\infty<x_1<\infty\\
&\begin{Bmatrix} \sigma_{21}(x_1,-h^{+}) \\ \sigma_{22}(x_1,-h^{+}) \end{Bmatrix} = \begin{Bmatrix} \sigma_{21}(x_1,-h^{-}) \\ \sigma_{22}(x_1,-h^{-}) \end{Bmatrix}, \quad -\infty<x_1<\infty,
\end{aligned} \tag{1}$$

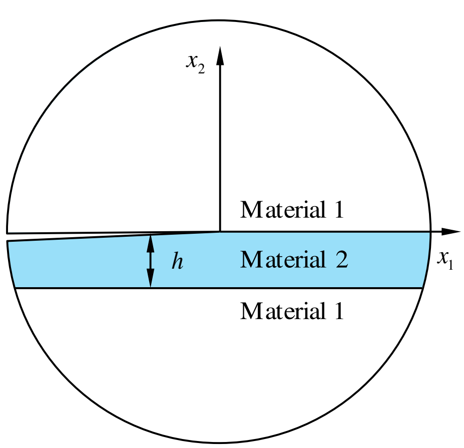

Fig. 1. Asymptotic problem of an interface crack in an orthotropic sandwich structure.

where σ_{ij} and u_i represent the stress and displacement, respectively. We restrict our attention to inplane deformations of the orthotropic solids under plane strain or plane stress conditions. The relations between the strains and stresses for the inplane deformation can be written as (Lekhnitskii, 1963)

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} S^e_{11} & S^e_{12} & 0 \\ S^e_{12} & S^e_{22} & 0 \\ 0 & 0 & S^e_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}, \tag{2}$$

where ε_{ij} is the strain, and $S^e_{ij} = S_{ij}$ for plane stress and $S^e_{ij} = S_{ij} - S_{i3}S_{j3}/S_{33}$ for plane strain in which S_{ij} is the conventional compliance component. A general solution to the equilibrium equation for the displacements and the corresponding stresses can be written in terms of two analytic functions as (Eshelby et al., 1953; Stroh, 1958)

$$\begin{aligned}
\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} &= 2\mathrm{Re}\left[\mathbf{A}\begin{Bmatrix} f_1(z_1) \\ f_2(z_2) \end{Bmatrix}\right],\\
\begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \end{Bmatrix} &= -2\mathrm{Re}\left[\mathbf{B}\begin{Bmatrix} p_1 f_1'(z_1) \\ p_2 f_2'(z_2) \end{Bmatrix}\right],\\
\begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \end{Bmatrix} &= 2\mathrm{Re}\left[\mathbf{B}\begin{Bmatrix} f_1'(z_1) \\ f_2'(z_2) \end{Bmatrix}\right].
\end{aligned} \tag{3}$$

Here, Re denotes the real part and $()'$ indicates the derivative with respect to the associate argument. Functions $f_j(z_j)$ $(j = 1, 2)$ are analytic in their arguments, and $z_j = x_1 + p_j x_2$ wherein the characteristic roots p_j $(j = 1, 2)$ (Suo, 1990b) are given by

$$\begin{aligned}
p_1 &= i\lambda^{-\frac{1}{4}}(n+m),\\
p_2 &= i\lambda^{-\frac{1}{4}}(n-m),
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
n &= \sqrt{\frac{1}{2}\left(\rho+1\right)},\\
m &= \sqrt{\frac{1}{2}\left(\rho-1\right)}.
\end{aligned} \tag{5}$$

λ and ρ are dimensionless parameters defined by

$$\begin{aligned}
\lambda &= \frac{S^e_{11}}{S^e_{22}},\\
\rho &= \frac{2S^e_{12}+S^e_{66}}{2\sqrt{S^e_{11}S^e_{22}}}.
\end{aligned} \tag{6}$$

The parameters λ and ρ, which characterize the orthotropy of the material, are restricted to take on the values $\lambda > 0$ and $-1 < \rho < \infty$, respectively, due to the positive definiteness of the strain energy density. In particular, $\lambda = \rho = 1$ for an isotropic material. The components of matrices $\mathbf{A}$ and $\mathbf{B}$ for orthotropic material may be written as (Stroh, 1958; Suo, 1990b)

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} S^e_{11}p_1^2 + S^e_{12} & S^e_{11}p_2^2 + S^e_{12} \\ S^e_{12}p_1 + \dfrac{S^e_{22}}{p_1} & S^e_{12}p_2 + \dfrac{S^e_{22}}{p_2} \end{bmatrix},\\
\mathbf{B} &= \begin{bmatrix} -p_1 & -p_2 \\ 1 & 1 \end{bmatrix}.
\end{aligned} \tag{7}$$