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A new fatigue failure theory for multidirectional fiber-reinforced composite laminates with arbitrary stacking sequence

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A B S T R A C T

Fatigue failure is one of the most important failure types of fiber-reinforced composites. In this paper, a new fatigue failure theory for multidirectional fiber-reinforced composite laminates with an arbitrary stacking sequence is developed, by combining nonlinear residual strength and residual stiffness models with the recently improved Puck's failure theory which includes the *in situ* strength effect. This fatigue theory can predict the fatigue life, residual strength and residual failure envelope of fiber-reinforced composite laminates under multidirectional loadings. For these predictions it is necessary to recalculate the fatigue lives of laminae after each cycle since the stresses in the laminae change due to stiffness degradation. It is also necessary to account for the nonlinear accumulation of damage at the new stress level in the laminae resulting from stiffness degradation. This is achieved by using the concept of equivalent cycle. The theoretical predictions are in good agreement with available experimental results.

1. Introduction

Theoretical prediction of the failure of composite laminates is of great importance in order to make the best use of them. Fatigue failure is one of the most important failure types of fiber-reinforced composites during their service. Due to inhomogeneity and anisotropy, fatigue failure mechanisms of fiber-reinforced composites are vastly different from their counterparts in homogeneous and isotropic materials, like metals. For example, there exist a variety of failure types in the composites, including matrix cracking, interfacial debonding, delamination and fiber rupture, while the failure type of metals is relatively simple. This complexity makes it difficult to develop a robust and efficient fatigue failure theory for fiber-reinforced composites.

The current fatigue failure theories for unidirectional lamina can be classified into three different types [1]. The first type is the fatigue life theory which usually uses *S–N* curves or Goodman-type diagrams, but the actual failure mechanisms are not taken into account. The second type is the phenomenological theory, mainly including residual stiffness and strength models. The third type are the progressive damage models which consider the local failure mechanisms and usually apply one or more damage variables related to the measurable manifestations of damage such as the transverse matrix cracking or the delamination. The details of these models can be found in [2–9], among others.

The fatigue failure theories for multidirectional laminates can be divided into two categories. The first are the theories that can only be applied to laminates of specific configuration. For example, Rotem and Hashin [10] developed a fatigue failure theory based on the Hashin–Rotem static failure criterion, which has been used to predict the fatigue failure of angle-ply laminates. Kawai and co-workers [11,12] developed a fatigue failure theory based on the Tsai-Hill static failure criterion, which has also only been applied to prediction of the fatigue failure of angle-ply laminates. The second category are the theories that can be generally applied to laminates of any configuration. For example, Shokrieh and Lessard [13,14] proposed a fatigue failure theory for multidirectional laminates by incorporating the modified interacting residual strength and stiffness models into failure criteria for seven different kinds of damage. However, they do not include the *in situ* strength effect. Passipoularidis et al. [15] developed a fatigue theory based on Puck's static failure criterion, but they do not consider the nonlinear effect of strength degradation and the *in situ* strength effect. Other research on the fatigue failure of fiber-reinforced composites can be found in [16–24], among others.

Recently, an improved Puck's failure theory [25] for static failure of composite laminates has been developed, by combining

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the original Puck's failure theory [26–28] with the fracture mechanics based *in situ* strength theory developed by Wang and Karihaloo [29]. In this paper, a new fatigue failure theory for multidirectional fiber-reinforced composite laminates with an arbitrary stacking sequence is developed, by combining nonlinear residual strength and residual stiffness models with the improved Puck's failure theory. Compared with the existing fatigue failure theories discussed above, this theory is novel in the following respects. Firstly, the *in situ* strength effect is physically included, as the fatigue theory is based on the improved Puck's static failure theory. Secondly, the nonlinear degradation of both the strength and stiffness is taken into account through residual strength and stiffness models. Thirdly, this fatigue theory can predict the fatigue life, residual strength and residual failure envelopes of multidirectional fiber-reinforced composite laminates with an arbitrary stacking sequence under multidirectional loadings. For these predictions it is necessary to recalculate the fatigue lives of laminae after each cycle since the stresses in the laminae change due to the stiffness degradation. It is also necessary to account for the nonlinear accumulation of damage at the new stress level in the laminae resulting from the stiffness degradation. This is achieved by using the concept of equivalent cycle. The predictions of the theory are shown to be in good agreement with available experimental results.

2. A brief review of the improved Puck's failure theory

Just like the original Puck's failure theory [26–28], the improved version [25] also contains two failure types: the fiber failures and inter-fiber failures. The fiber failures include two different failure modes defined by

$$\frac{\sigma_1}{X_t} - \nu_{12}\frac{\sigma_2}{X_t} + \nu_{f12}\frac{E_1}{E_2}\cdot m_{\sigma f}\frac{\sigma_2}{X_t} = 1, \quad \text{for } \sigma_1 > 0, \tag{1}$$

$$\left|\frac{\sigma_1}{X_c} - \nu_{12}\frac{\sigma_2}{X_c} + \nu_{f12}\frac{E_1}{E_2}\cdot m_{\sigma f}\frac{\sigma_2}{X_c}\right| + (10\gamma_{12})^2 = 1, \quad \text{for } \sigma_1 < 0, \tag{2}$$

where $\{X_t, X_c\}$ are the longitudinal tensile and compressive strengths of a single lamina, $\{E_1, E_{f1}\}$ are the longitudinal moduli of the lamina and the fiber, ν_{12} is the Poisson ratio of the lamina, ν_{f12} is the Poisson ratio of the fiber which is defined as the ratio of the strain in the transverse direction caused by a stress in the longitudinal direction to the strain in the longitudinal direction caused by the same stress, and $\{\sigma_1, \sigma_2\}$ are the longitudinal and transverse normal stresses of the lamina. The parameter γ_{12} is the in-plane shear strain of the lamina. The factor $m_{\sigma f}$ accounts for the 'stress magnification effect' caused by the mismatch between the moduli of fibers and matrix.

As to the inter-fiber failures, there are in all three different modes: Mode A, Mode B and Mode C. The corresponding failure criteria are.

Mode A:

$$\sqrt{\left(\frac{\tau_{12}}{S_{12}^I}\right)^2 + \left(1 - p_{\perp\parallel}^{(+)}\frac{Y_t^I}{S_{12}^I}\right)^2\left(\frac{\sigma_2}{Y_t^I}\right)^2} + p_{\perp\parallel}^{(+)}\frac{\sigma_2}{S_{12}^I} = 1 - \left|\frac{\sigma_1}{\sigma_{1D}}\right|, \quad \sigma_2 \geqslant 0; \tag{3}$$

Mode B:

$$\frac{\sqrt{\tau_{12}^2 + \left(p_{\perp\parallel}^{(-)}\sigma_2\right)^2} + p_{\perp\parallel}^{(-)}\sigma_2}{S_{12}^I} = 1 - \left|\frac{\sigma_1}{\sigma_{1D}}\right|, \quad \sigma_2 < 0 \text{ and } 0 \leqslant \left|\frac{\sigma_2}{\tau_{12}}\right| \leqslant \frac{R_{\perp\perp}^A}{\tau_{12c}}; \tag{4}$$

Mode C:

$$\left[\left(\frac{\tau_{12}}{2(1+p_{\perp\perp}^{(-)})S_{12}^I}\right)^2 + \left(\frac{\sigma_2}{Y_c}\right)^2\right]\frac{Y_c}{-\sigma_2} = 1 - \left|\frac{\sigma_1}{\sigma_{1D}}\right|, \quad \sigma_2 < 0 \text{ and } 0 \leqslant \left|\frac{\tau_{12}}{\sigma_2}\right| \leqslant \frac{\tau_{12c}}{R_{\perp\perp}^A}; \tag{5}$$

where $\{\sigma_2, \tau_{12}\}$ denote the transverse normal stress and in-plane shear stress. The term $|\sigma_1/\sigma_{1D}|$ represents the degradation of the resistance in the inter-fiber fracture conditions due to single fiber failure. The parameters $\{p_{\perp\parallel}^{(+)}, p_{\perp\parallel}^{(-)}, p_{\perp\perp}^{(-)}\}$ are constants. For glass-fiber reinforced composites, the recommended values of $\{p_{\perp\parallel}^{(+)}, p_{\perp\parallel}^{(-)}\}$ are $\{0.3, 0.25\}$, and for carbon-fiber reinforced composites are $\{0.35, 0.3\}$ [28]. The value of $p_{\perp\perp}^{(-)}$ can be calculated from

$$p_{\perp\perp}^{(-)} = \frac{1}{2}\left[\sqrt{1 + 2p_{\perp\parallel}^{(-)}\frac{Y_c}{S_{12}}} - 1\right]. \tag{6}$$

The parameters $\{Y_t^I, S_{12}^I\}$ in Eqs. (3)–(5) denote the *in situ* values of the transverse tensile and in-plane shear strengths of a specific lamina embedded in the laminate. Y_c denotes the transverse compressive strength of the lamina, which has no *in situ* effect. $\{Y_t^I, S_{12}^I\}$ can be expressed by [29]

$$Y_t^I = Y_t\left[1 + \frac{A}{N^B}f_t(\Delta\theta)\right], \tag{7}$$

$$S_{12}^I = S_{12}\left[1 + \frac{C}{N^D}f_s(\Delta\theta)\right], \tag{8}$$

where $\{Y_t, S_{12}\}$ are the strengths of a lamina in isolation. The parameters $\{A, B, C, D\}$ are to be determined by experiments, and their dependence upon the laminate configuration is discussed by Wang and Karihaloo [30]. The parameter N denotes the number of plies of given thickness in a multidirectional laminate, which simply represents the influence of the lamina thickness. The two functions $\{f_t(\Delta\theta), f_s(\Delta\theta)\}$ represent the influence of the neighboring laminae on the strengths of an embedded lamina. They are given by

$$f_t(\Delta\theta) = \min\left[\frac{\sin^2(\Delta\theta_a)}{1+\sin^2(\Delta\theta_a)}, \frac{\sin^2(\Delta\theta_b)}{1+\sin^2(\Delta\theta_b)}\right], \tag{9}$$

$$f_s(\Delta\theta) = \min\left[\frac{\sin^2(2\Delta\theta_a)}{1+\sin^2(2\Delta\theta_a)}, \frac{\sin^2(2\Delta\theta_b)}{1+\sin^2(2\Delta\theta_b)}\right], \tag{10}$$

where $\{\Delta\theta_a, \Delta\theta_b\}$ denote the ply angle differences between the lamina under consideration and laminae immediately above and below it.

The three failure modes A, B, and C are schematically illustrated in Fig. 1. The Mode A failure is mainly caused by the transverse tensile stress, the Mode B failure by the in-plane shear stress, and the Mode C failure by the transverse compressive stress. The ratio of the parameters $R_{\perp\perp}^A$ and τ_{12c} determine the critical condition for the transition from Mode B to Mode C, when the lamina is under $\sigma_2 - \tau_{12}$ combined stress state. They are given by

$$R_{\perp\perp}^A = \frac{Y_c}{2(1+p_{\perp\perp}^{(-)})}, \quad \tau_{12c} = S_{12}^I\sqrt{1+p_{\perp\perp}^{(-)}}. \tag{11}$$

The left hand terms of Eqs. (3)–(5), composed of the stresses $\{\sigma_2, \tau_{12}\}$ and the strengths $\{Y_c, Y_t^I, S_{12}^I\}$, are denoted by f_E – the 'stress exposure factor'. This factor describes the proximity of the current stress state of the lamina to the failure state. The lamina is identified as not having failed if $f_E < 1$ and it is regarded as having failed by inter-fiber failure when the value of this factor reaches $f_E = 1$ (when $\sigma_1 = 0$).