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A stress versus crack growth rate investigation (aka stress – cubed rule)

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ABSTRACT

A means of determining the effect on the crack growth rate of a variation in the stress level, for the same basic spectrum and material, is often needed for the interpretation of aircraft fatigue test results, for the design of repairs or to assess the effect of weight increases. This paper describes one such tool colloquially known as the stress-cubed (or cubic) rule, and provides examples of its application to a number of materials, spectra and stress concentrations. It is shown that for lead cracks the CG rate at one stress level can be predicted accurately with knowledge of the second stress level, its CG rate and the effective initiating crack size.

1. Introduction

A means of determining the effect on the crack growth rate of a variation in stress level, for the same basic spectrum, is often needed for fatigue analyses. One such tool is colloquially known as the stress-cubed (or cubic) rule and is used by the Royal Australian Air Force in the Hornet Structural Analysis Methodology (SAM) [1] and the P3C Repair Assessment Methodology (RAM) [2].

For a given spectrum, the relative crack growth rates associated with cracks that grow from small naturally occurring material discontinuities for varying stress levels can often be estimated using the Frost and Dugdale model [3]. The Frost and Dugdale model,[1] which was formulated for constant amplitude data, postulated that the exponent representing the rate of exponential crack growth [4], ψ, could be expressed as a function of the applied stress:

$$\psi = \sigma^{\alpha}\lambda \tag{1}$$

where σ is the applied constant amplitude stress and α and λ are empirical constants.

Refs. [4,6–11] extended the use of the Frost and Dugdale model to variable amplitude loading, by relating the crack growth rate exponent to a reference stress, σ_{REF}, that was related to the spectrum[2]:

$$\psi = \sigma_{REF}^{\alpha}\lambda \tag{2}$$

This extended Frost–Dugdale rule states that crack growth is exponential and can be written as:

$$a = a_0 \exp\left(\sigma_{REF}^{\alpha}\lambda\, t\right) \tag{3}$$

where t is time, a_0 is the initial effective crack depth at time $t = 0$, and a is the crack depth at time t.

The crack growth rate can be then be expressed as:

$$da/dt = a_0\sigma_{REF}^{\alpha}\lambda \quad \exp\left(\sigma_{REF}^{\alpha}\lambda\, t\right) = a\,\sigma_{REF}^{\alpha}\lambda \tag{4}$$

$$(da/dt)/a = \sigma_{REF}^{\alpha}\lambda \tag{5}$$

According to this relationship, at a given crack length the ratio of crack growth rates for two tests performed under the same spectrum (i.e. λ is constant), but at two different reference stress levels, can be expressed as:

$$(da/dt)_1/(da/dt)_2 = \left(\sigma_{REF.1}^{\alpha}\lambda\right)/\left(\sigma_{REF.2}^{\alpha}\lambda\right) = \left(\sigma_{REF.1}/\sigma_{REF.2}\right)^{\alpha} \tag{6}$$

where the subscripts 1 and 2 refer to reference stress levels, i.e. stress levels 1 and 2 respectively.

As was observed by Frost and Dugdale for constant amplitude loading, a value of $\alpha \approx 3$ was found to apply for a range of materials tested under variable amplitude loading with a low stress concen-

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[1] Frost and Dugdale's work was limited to thin sheets of mild steel (3.25 mm thick), aluminium alloy (2 mm) and copper (3.25 mm).

[2] Judgement must be exercised when choosing $\sigma_{REF.}$ For instance choosing the maximum stress in a spectrum with a single large overload would not be appropriate.

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tration factor (i.e. $K_t \approx 1.0$) [4,6–8,10].[3] Thus, the model is hitherto referred to as the stress-cubed rule.

This paper, which focuses on the application to aerospace related problems, considers a further range of materials and as only limited investigations exist [9,11], specifically considers high K_t configurations. It is shown that the cubic is applicable to high K_t configurations as long as net section yield does not occur. This is an important finding and has the potential to broaden the application of the cubic rule.

2. The lead crack framework

The cubic rule is only applicable to lead cracks [12]. For the purposes of the analyses presented here, the following caveats apply to lead cracks:

(1) They start to grow shortly after testing begins or the aircraft is introduced into service.
(2) They grow approximately exponentially, subject to caveats which include [12] the absence of significantly varying finite width correction factors (i.e. up to the point of a significant change in geometry occurs).
(3) They commence growing from small (sub-millimetre) discontinuities that are related to the near surface quality of the production material.
(4) The small fraction of the crack growth life influenced by quasi-static fracture close to final failure is ignored.

To apply the cubic rule in practice, at least one set of crack growth data (i.e. crack size versus loading history), preferably obtained down to small crack sizes (through quantitative fractography), must be available.

For the examples provided in the following sections the size of the initiating discontinuity is generally provided or readily found from the back-projection of the crack growth data available (note: for both the source and the to-be-predicted data). In cases where the initial discontinuity size is not apparent from the available crack growth data, an estimate of the Equivalent Pre-crack Size (EPS) as defined in [5,14] will be required. For practical solutions a mean or a "specified number of standard deviations from the mean" (for an acceptable probability of failure) EPS will be required for the specific initial discontinuity (which is a function of surface finish and or manufacturing details), see for example [15].

In this context it should be noted that as explained in ASTM fatigue test standard E647-13a and [12,16]:

"Fatigue cracks of relevance to many structural applications are often small or short for a significant fraction of the structural life." It follows that:

As such the most important (critical) region for the determining the exponent is the sub-mm region. It also follows that the data analysed should be associated with the lead cracks [12], i.e. the fastest growing cracks in the structure. In this paper the fastest crack is taken to be that which has the largest crack growth rate, i.e. has the largest value of ψ.

In such cases [16]: notes that the fatigue threshold is small. Indeed, ASTM fatigue test standard E647-13a notes that:

> "It is not clear if a measurable threshold exists for the growth of small fatigue cracks ...."

Lead cracks are distinguished from non-lead cracks by the characteristic (amongst others, see [12]) that the associated fatigue thresholds are very small.

[3] This finding may explain the statement in Def Standard 970 [13] Section 4, that "damage is proportional to the cube of the amplitude".

Finally, the cubic rule is only applicable for the estimation of crack growth between two locations with similar[4] stress concentration factors.

3. Some coupon fatigue test results

3.1. Crack growth from a hole under a F4E/S flight load spectrum

To further illustrate that da/dt is proportional to the cube of the stress consider the data by Potter et al. [17], who presented crack growth data for 0.5 in. (12.7 mm) thick, and 1 in. (25.4 mm) wide aluminium alloy (AA) 7075-T6511 specimens with a working length of 6.5 in. (165 mm). These specimens contained a centrally located 0.25 in. (6.35 mm) diameter hole that was notched on one side to start a corner crack. These specimens were designed so as to obtain the propagation behaviour of corner cracks growing out of holes, which is a typical problem in aircraft structures. In this study the spectrum used was derived from the bending moment spectrum at Load Reference Station (LRS) 140 of the McDonnel Douglas F-4E Slatted Wing fatigue test aircraft and contained 320 air-to-ground, 230 air-to-air, and 180 non-tactical flights per 1000 flight hours. Two tests were performed; one with a remote stress of 30 ksi (207 MPa) and another at a remote stress of 36 ksi (248.2 MPa) i.e. a change in stress range of 20%. The associated crack length histories are shown in Fig. 1.

Barter et al. [4] used the cubic rule to predict the crack depth history at the higher (248 MPa) stress level from the lower stress level (207 MPa) and the resultant prediction is also shown in Fig. 1 where we see good agreement between the predicted and measured crack length histories. This prediction used the initial exponential slope of the a versus N curve to compute the crack growth histories at different stress levels. Since this was the first application of the cubic rule to predict crack growth histories at different stress levels we will term this approach "Method 1". Note an estimate of the initial effective crack size (e.g. [5,14]) for each crack growth curve is required to perform the prediction (note: this applies to all predictions in this paper). In this example (as in all others) the crack growth was extrapolated back to zero flight hours to define the EPS (see constant in fitted equation) for both the source (stress level 1) and the predicted (stress level 2).

The conclusions that follow from this study are that:

(1) Whilst the crack length versus flight hours curves are approximately log-linear (i.e. crack growth is exponential) the exponent ψ in Eq. (1), i.e. the factor in exponential crack growth equation, can differ significantly depending on whether all of the data points are considered or whether only the initial exponential (in the case of the 207 MPa test data points up to approximately 1 mm) crack growth history is considered. Indeed, the exponent ψ determined from the initial exponential crack lengths (Method 1) is approximately 50% larger than that associated with the entire crack length histories which we termed Method 2. For the 248.2 MPa tests Method 1, i.e. where the exponent ψ is determined from the sub-1 mm crack length history, gave $\psi = 0.0031$ compared to a value of $\psi = 0.0018$ determined using Method 2, i.e. where the exponent ψ is determined from the entire crack length history. For the 207 MPa tests Method 1 gave $\psi = 0.0018$ compared to a value of $\psi = 0.0011$ determined using Method 2. The difference in the value of ψ obtained using these two different methods is due to the drop in the stress away from the hole, i.e. the decay of the β function with crack length.

[4] Further analyses are required to better understand the range of applicability.