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The Laplace multi-axial response model for fatigue analysis

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ABSTRACT

This paper introduces means for fatigue damage rates estimation using Laplace distributed multiaxial loads. The model is suitable for description of stresses containing transients of random amplitudes and locations. Explicit formulas for computing the expected value of rainflow damage index as a function of excess kurtosis are given for correlated loads. A Laplace model is used to describe variability of forces and bending moments measured at some location on a cultivator frame. An example of actual cultivator data is used to illustrate the model and demonstrate the accuracy of damage index prediction.

1. Introduction

Stochastic modeling of loads is usually done with stationary Gaussian processes. Well-developed numerical tools for computing the probabilities of interests are available, see e.g. [1]. However, many of the environmental loads that act on for example ground vehicles are far from being Gaussian. Nevertheless, Gaussian models are often used, and this sometimes leads to serious underestimation of risks for fatigue.

Estimations of component durability often requires a customer or market specific load description. One is interested in having models that are capable of describing the correct variability of loads with a relatively small number of parameters. These models can then be used to describe the long term loading by means of a distribution of the parameter values, specific for a given market or encountered by specific customers.

The severity of environmental loads can be measured by means of damage indexes. In the case when Gaussian models are used for stresses, there are many methods for estimating the expected damage indexes from the power spectrum density, psd, see eg. [2] for a review of different approaches. Much less is known for loads containing transients. In this paper, explicit formulas for computing expected value of rainflow damage index as a function of excess kurtosis will be given, for a special case of equally distributed although correlated Laplace loads.

A Gaussian model can be seen as the result of smoothing Gaussian white noise, i.e. a sequence of independent standard Gaussian variables, by a suitable kernel. When a cultivator is operating in light sandy soils where stones are frequent, the vibrations have a larger spread of variation that cannot be modeled by solely Gaussian processes. The Laplace white noise is used to model the larger spread by letting Gaussian white noise have variable variance. This is achieved by multiplying the Gaussian variables by the square root of gamma distributed factors. The factors have mean value one, and hence, loads derived by smoothing Gaussian or Laplace noise with the same kernel will have identical power spectrum densities (psd). However, in contrast to the Gaussian process, the Laplace process will have visible transients at times when factors take large values.

In this paper, we present models for loads, which are forces and bending moments, measured at some point of a stiff mechanical structure. For example, the method is used to asses the durability of welds in a stiff frame of a cultivator. Hence accurate description of stress variability at welds are needed. For a stiff frame, stresses are linear combinations of environmental loads. This property makes modeling using Gaussian processes very convenient, since linear combinations of Gaussian loads are Gaussian processes as well and any probability of interest can in principle be computed when the psd of the loads are available.

In Fig. 1, six loads, three forces and three moments measured on one tine, are presented. One can see that transients appearing in

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Nomenclature

a	spectrum scale parameter (dimensionless)
β	damage exponent (dimensionless)
$\mathbf{c} = (c_1, \ldots, c_M)$	constants combining loads into stress: $10^6 \times [\mathrm{m}^{-3}]$ (bending moments), $10^6 \times [\mathrm{m}^{-2}]$ (forces)
$D(\mathbf{c}) = D_{\mathbf{X}}(\mathbf{c})$	rainflow multiaxial damage index (m^β/s)
$D^{obs}(\mathbf{c})$	observed multiaxial damage index (m^β/s)
$\mathcal{D}(\mathbf{c}) = \mathcal{D}_{\mathbf{X}}(\mathbf{c})$	expected damage index (m^β/s)
F_x, F_y, F_z	forces in the principal directions (N)
$\mathcal{F}, \mathcal{F}^{-1}$,	Fourier transform and its inverse
$g(t)$	kernel for scale standardized moving averages
$\Gamma(\cdot)$	gamma function
$h_k^{rf}(\mathbf{c}),\ k = 1, \ldots, K$	the rainflow cycle ranges (m)
$\kappa,\ \kappa_e$	kurtosis and excess kurtosis of a load (dimensionless)
$M_x,\ M_y,\ M_z$	bending moments in the principal directions (Nm)
v	shape parameter in gamma distribution
ω	angular frequency (rad)
$\mathbf{R} = [R_i]$	gamma white noise
ρ	correlation in bivariate noise $[\mathbf{W}_1, \mathbf{W}_2]$
σ^2	variance of the load ($\mathrm{N}^2\,\mathrm{m}^2$) or ($\mathrm{N}^2$)
Σ	covariance matrix of the bivariate load ($\mathrm{N}^2\,\mathrm{m}^2$) or ($\mathrm{N}^2$)
$S_a(\omega)$	spectrum of bending moment load ($\mathrm{N}^2\,\mathrm{m}^2$/rad)
$S(\omega)$	normalized spectrum of a load (rad^{-1})
t, T	running time, total time, respectively (s)
$\mathbf{X}(t) = (X_1(t), \ldots, X_M(t))$	vector of loads: bending moments (Nm), forces (N)
$\mathbf{X}^{obs}(t)$	observed loads: bending moments (Nm), forces (N)
$X(t)$	scale normalized dimensionless load
$Y_{\mathbf{c}}(t)$	stress (MPa)
$\mathbf{W} = [W_i]$	white noise (independent equally distributed random variables)
$[\mathbf{W}_1, \mathbf{W}_2]$	white noise in the bivariate case
$\mathbf{Z} = [Z_i]$	Gaussian white noise

different forces and bending moments are often close in time. Since stress is a linear combination of the loads this may result in very large stresses which may greatly amplify the damage accumulation rate. The proposed multiaxial Laplace model for load will have the property of high frequency of simultaneous occurrences of large transients. Table 1 shows statistics for the dominating signals including the observed damage as defined in the next section.

2. Fatigue damage

In this paper, multivariate random processes $\mathbf{X}(t) = (X_1(t), \ldots, X_M(t))$ are used to represent multi-axial loads, being forces and bending moments acting on a structure at different locations. For a stiff structure, stresses, used to predict fatigue damage, are linear combinations of forces and moments. For this reason, it is important to model the multi-axial load so that a stress, i.e. a linear combination of loads

$$Y_{\mathbf{c}}(t) = \sum_{r=1}^{M} c_r X_r(t), \qquad t \in [0, T], \tag{1}$$

yields accurate fatigue accumulation. In the examples in this paper we focus on the situation where the sum above is over forces and moments measured at one position. If there are forces and moments in several positions, the sum should be over all of them. Since the vector $\mathbf{c} = (c_1, \ldots, c_M)$ may vary between locations in a structure experiencing the same loads $\mathbf{X}(t)$ one requires good accuracy for any choice of constants $\mathbf{c}$. These are typically evaluated using finite elements method and depend on geometry and material properties and transfer external loads to stresses at a point in the structure of interest. The fatigue damage accumulated in the material is expressed using a fatigue (damage) index defined by means of the rainflow method which is computed in the following two steps. First, rainflow ranges $h_k^{rf}(\mathbf{c}),\ k = 1, \ldots, K$, in $Y_{\mathbf{c}}(t)$ are found. Here K is the number of rainflow cycles which equals the number of local maxima. Then the rainflow damage is computed according to Palmgren–Miner rule [3,4], viz.

$$D(\mathbf{c}) = \frac{1}{T} \sum_{k=1}^{K} \left(h_k^{rf}(\mathbf{c})\right)^{\beta}, \tag{2}$$

see also [5] for details of this approach. Various choices of the damage exponent β can be considered. The value is an empirical constant estimated by means of regression from experiments involving constant amplitude loads. In this paper $\beta = 3$, which is the standard value for the crack growth process in a welded frame. The index $D(\mathbf{c})$ is often called multi-axial damage index and was introduced in [6].

The proposed model for the multi-axial loads $\mathbf{X}(t)$ is validated by using measured loads and comparing the ensuing damage index with the expected value of the damage index following from the model fitted to the data. In this, first the model parameters are estimated using measured loads $\mathbf{X}^{obs}(t)$. Then the expected theoretical damage index $\mathcal{D}(\mathbf{c}) = \mathsf{E}[D(\mathbf{c})]$ is estimated by means of Monte Carlo (MC) method and compared with $D^{obs}(\mathbf{c})$ for a suitably chosen vector of factors $\mathbf{c}$, where $D^{obs}(\mathbf{c})$ is computed by means of (2) with rainflow ranges obtained in the observed records. In our notation, we do not explicitly indicate that the expected damage index $\mathcal{D}(\mathbf{c})$ depends also on the properties and defining parameters of the process $\mathbf{X}$. In what follows, whenever this dependence needs to be exhibited, we write $D_{\mathbf{X}}(\mathbf{c})$ and $\mathcal{D}_{\mathbf{X}}(\mathbf{c})$ for the damage and the expected damage, respectively.

3. Uniaxial load

Power spectral density (psd) is an important characteristics of stationary stress (load). For Gaussian stress the fatigue damage index is a function of psd alone. Even for Laplace processes, psd remains an important characteristic. However, it in general does not determine the damage index completely. In this paper, a very simple, yet often used, reparametrization of the model for psd is used

$$S_a(\omega) = \sigma^2 a S(a\omega) \qquad a > 0, \tag{3}$$

where $\int S(\omega)\, d\omega = 1$, σ^2 is the variance of the load, while a is a spectrum scale parameter. A load with psd given in (3) can be written as

$$X_a(t) = \sigma X(t/a), \tag{4}$$

where $X(t) = X_1(t)/\sigma$, having psd $S(\omega)$, is a scale normalized load. The psd (3) and process (4), where $X(t)$ is Laplace moving average, have found applications, for example, in road roughness classifications, where a is the velocity a vehicle travels while $S(\omega)$ depends on the linear filter that has been used to model responses and the spectral properties of a road profile, see [7].

The proposed model is applicable to an arbitrary form of spectrum $S(\omega)$. For modeling cultivator loads, the following psd proves to be useful

$$S(\omega) = 0.5 \exp(-|\omega|), \tag{5}$$