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How proper similitude can improve our understanding of crack closure and plasticity in fatigue

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ABSTRACT

The appropriateness of some common similitude principles with respect to describing and predicting fatigue damage propagation is discussed. Linear elastic fracture mechanics have provided a basis to describe damage growth using stress intensity factors or strain energy release rates, both related to the work of Griffith and Irwin. The fatigue crack growth equations presented in the literature are discussed, and it is demonstrated that the principles of similarity in current methodologies have not yet been well established. As a consequence, corrections for the stress ratio effect are misunderstood. An alternative principle of similitude using cyclic work and strain energy release is proposed.

1. Introduction

Although many researchers these days may not realize this explicitly, anywhere in the field of engineering and science, a principle of similarity is adopted. For example, to predict when quasi-static failure occurs in a complex 3-dimensional Finite Element simulation, often stresses are correlated under the assumption that similarity between stresses in the model and in a tensile specimen will yield similar consequences.

Scrutinizing the vast number of papers in the literature on fatigue damage growth, illustrates that most discussions on the validity of proposed methodologies narrow down to the question: what is the most appropriate principle of similitude? Once a principle of similitude has been agreed upon, most studies tend to follow this approach without further questioning the fundamentals underlying this principle [1]. From an engineering perspective this is preferred, because continuous questioning of basic principles will hinder progress in research and technology. However, from an academic or scientific perspective, one may expect continuous criticism with respect to fundamentals of selected principles of similarity.

In this paper the appropriateness of the principles of similitude currently adopted for fatigue damage growth within the context of linear elastic fracture mechanics is questioned. Various observations seem to indicate that currently trends are being misinterpreted simply because similitude has not been well established.

2. Reviewing current fatigue approaches

2.1. Stress and strain based fatigue approaches

Traditionally, mechanics of materials has been dealt with using stresses and strains. For most quasi-static loading conditions, this principle seems appropriate and has proven its usefulness in the field of science and engineering.

Once fatigue as a degradation or wear-out phenomenon was acknowledged [1], engineers and scientists initially approached the problem using these similarity principles at hand, i.e. engineering stresses and strains. The early days of fatigue research are characterized by studies and papers that propose approaches based on stress and strain. For example, August Wöhler proposed to plot the observed failure life against the stress amplitude [1], because he regarded this as being most decisive for the destruction of material cohesion. According to him, the maximum stress is of influence only in so far as the higher it is, the lower is the stress amplitude which leads to failure. This principle of similitude has never really been questioned and most engineering handbooks [2–4] presently utilize these *S*–*N* curves for design.

2.2. Crack propagation approaches

At some point, a distinction was made between the phases of fatigue. The first phase covers the nucleation and propagation of microscopically small cracks, while the second phase covers the propagation of macroscopically sized cracks [5].

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Nomenclature

A	area, or crack surface (mm^2)	P	applied load (N)
a	crack length (mm)	P	pressure (Pa)
C	correction factor or constant	Q	heat (mJ)
ε	strain (–)	R	stress ratio S_{min}/S_{max} (–)
G	strain energy release rate (N/mm)	r_p	plastic zone size (mm)
γ_e	surface energy per unit area (mJ/mm^2)	S	global stress (MPa)
K	stress intensity factor ($MPa\sqrt{mm}$)	S_y	yield strength (MPa)
K_t	stress concentration factor (–)	U	strain energy (mJ)
L	length (mm)	V	volume (mm^3)
m	exponent	Y	crack closure correction (–)
N	number of load cycles (cycles)		
n	exponent		

Crack propagation required a different approach compared to the evaluation of the fatigue initiation life. People attempted to relate the rate of propagation to a variety of parameters representing similarity. Hence, various crack propagation equations were proposed, which erroneously often are referred to as crack propagation laws, like for instance 'the Paris law' [6–9]. The field of crack growth description is characterized by an engineering approach rather than a scientific approach, as illustrated by the many corrections to parameters describing the conditions. Thus it is the author's opinion that the word 'law' should be considered highly inappropriate in this field in particular.

The first well known crack growth relation was proposed by Head [10,11], which was based upon a mechanical model using rigid-plastic work hardening assuming a constant plastic zone size. After correction for the increase in plastic zone size proportional to the crack length [12], this equation was modified to [13].

$$\frac{da}{dN} = \frac{CS^2 a}{S_y - S} \quad (1)$$

Frost and Dugdale [12] observed that the propagation of cracks in metallic materials seem to correlate to the cube of the stress rather than its square, i.e.

$$\frac{da}{dN} = \frac{S^3 a}{C} \quad (2)$$

McEvily and Illg [14] proposed another formulation based upon a fictitious crack tip radius and using the stress concentration factor K_t, resulting in

$$\frac{da}{dN} = K_t S_{nom} \quad (3)$$

with K_t obviously a function of the given crack tip radius. Meanwhile, Paris et al. [15] proposed to adopt Irwin's [16] Stress Intensity Factor (SIF) K, arguing that this parameter reflects the influence of external load and geometry. His relation can be formulated as

$$\frac{da}{dN} = C\Delta K^n = C(\Delta S\sqrt{\pi a})^n \quad (4)$$

where n is commonly between 2 and 4 for metallic materials. Frost et al. [17] reanalysed existing data with Eq. (4) and concluded that it was less satisfactory than Eq. (2). Another formulation was proposed by Liu [18,19] who hypothesized that the saturation of hysteresis energy absorbed by the material during every cycle could be used as a criterion. The resulting formulation may be written as

$$\frac{da}{dN} = CS^2 a \quad (5)$$

Paris and Erdogan discussed these crack growth formulations in detail in [13], but in their discussion they seem to suggest that correlation between empirical relation and data validates the empirical relation. Obviously, all above empirical crack growth relations correlate to data, but that is simply because of their empirical nature. However, both authors correctly conclude in [13] that more data should be employed to verify whether any of these formulations is appropriate.

Reviewing the above equations, one observes that the general observation of all of the above mentioned authors is that the crack propagation rate correlates to the applied stress and the crack length according to

$$\frac{da}{dN} \sim S^n a^m \quad (6)$$

where n may range between 2 and 4, and m between 1 and 2. But the fundamental question to be asked is: What does this mean?

2.3. Capturing the cyclic nature

What may be observed reviewing the above referenced literature is the ambiguous use of either S or ΔS. Where, for example, originally the relation was proposed in terms of S, other papers refer to the original relation while using ΔS. In the end, it seems that generally it is deemed appropriate to use ΔS to represent fatigue crack growth, which seems in agreement with the original stress based approaches for fatigue life that describe the fatigue life using the stress amplitude S_a.

2.4. On the stress intensity factor range ΔK for similitude

At a given point most people applied the SIF concept to describe similitude in fatigue crack propagation [20]. The SIF is generally referred to as 'the controlling variable for analyzing crack-extension rates' [21]. To describe the cyclic nature of fatigue loading, in general, the Paris relation given by Eq. (4) is adopted. However, the SIF range does not provide a comprehensive description of similitude, which is illustrated by the vast amount of data reported that show an apparent stress ratio effect [22–25].

Although state-of-the art, it seems incorrect to consider the SIF concept beyond any dispute. Various authors [26–32] have discussed the inappropriateness of using a single parameter ΔK to describe crack growth. Indeed, where the load (or stress) cycle requires 2 parameters to be described, it does not seem reasonable to assume that a single parameter equivalent to ΔS suffices to describe growth as result of that load cycle.

Consequently, two major lines of reasoning may be identified while reviewing the literature. Either an effective SIF range ΔK_{eff}