International Journal of Fatigue 66 (2014) 229–245

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

New energy model for fatigue life determination under multiaxial loading with different mean values

Krzysztof Kluger *, Tadeusz Łagoda

Opole University of Technology, 45-271 Opole, ul. Mikołajczyka 5, Poland

A R T I C L E I N F O

Article history:
Received 12 December 2013
Received in revised form 12 April 2014
Accepted 15 April 2014
Available online 24 April 2014

Keywords:
Multiaxial fatigue
Mean stress
Constant amplitude loading
Stress model
Energy model

A B S T R A C T

The paper proposes a new variant of the stress–strain parameter used to estimate the fatigue life in the multiaxial stress states with the influence of mean stress. The results of the fatigue life calculated according to the proposed variant have been compared to the results of fatigue tests of specimens made of 2017A-T4 aluminum alloy and S355J0 and 30NCD16 alloy steel in conditions of constant amplitude bending and torsion stress, as well as proportionate combinations of bending and torsion, while taking into account the mean stress. The experimental results have been compared to the results of calculations using models by Goodman, Gerber, Morrow, Findley, Dang Van, McDiarmid, Papadopoulos and Smith–Watson–Topper. Statistical analysis have been carried out for the results of calculations, involving the calculation of a scatter band for results of the comparison of experimental data with calculations.

1. Introduction

Despite the growing number of studies on materials fatigue and the growing interest of researchers in this issue, so far these studies failed to unequivocally develop an effective method to predict the degree of fatigue damage and a safe operational life of components, systems, as well as whole machines and structures. This is because fatigue phenomenon is very complex, and a fatigue destruction depends on many factors, such as the type and condition of the material, geometry of the element, type of load [1] or the state of stress [2]. Therefore research goes on, and with the development of knowledge about the fatigue of materials, certain narrow specializations separate that capture some fragments only of an extensive range of this subject matter, such as the accumulation of random loads, taking the mean values into account [3–10], analysis of the influence of the non-proportional load [11], determination of stresses and elastic–plastic strain, reduction of multiaxial stress state to an equivalent uniaxial state, etc. [3]. In case of uniaxial stress state, which exists in tension–compression, the calculation of fatigue life is reduced to only determine the function describing the dependence of the number of cycles on the load level [8–10]. In case of a multiaxial state that we are mostly dealing with in real structures or machine components, it is necessary to reduce this state to an equivalent uniaxial state. There are a number of hypotheses regarding fatigue, but none of them takes into account all the factors determining the development of fatigue damage. None of them is also universal enough to enable its use for any material, geometry and load conditions.

The criteria in the stress and strain notation usually do not take into account the response of the material to change in the path of the load and its effect on the fatigue process, which heavily depends on cyclic plastic deformations (strain), which in turn depend on the changing load path (the relationship between stress and strain). Therefore, they do not allow to obtain relevant results for cyclically unstable materials and for non-proportional loads. The solution may be to take into account both components of the stress and strain state in the process of determining the fatigue life by using the so-called energy notation. The additional advantage of this notation is that it can be used both in the low- and high-cycle range. The basis of the most of energy criteria and the derived fatigue life descriptions is the energy, which is permanently dissipated in the material under a variable load until the failure of the element, wherein the critical value of this energy determines the limit state of the material.

The non-zero mean value of stress is often the result of the effect of the working element's deadweight or the entire structure; it is also the result of the initial tension of load-bearing elements (such as V-belts in transmissions). The mean stress includes residual stresses arising as a result of joining of materials [12].

The purpose of this paper is to present the energy models (two variants of stress–strain parameter, Smith–Watson–Topper [13]) and stress models (Goodman [9], Gerber [8] Morrow [10] Findley

* Corresponding author.
E-mail addresses: k.kluger@po.opole.pl (K. Kluger), t.lagoda@po.opole.pl (T. Łagoda).

http://dx.doi.org/10.1016/j.ijfatigue.2014.04.008
0142-1123/

Nomenclature

A'	regression constant of the fatigue curve (for bending) scaled to the energy range
A_σ	regression constant of the fatigue curve (for bending)
A_τ	regression constant of the fatigue curve (for torsion)
b	fatigue strength exponent
c	fatigue ductility exponent
E	Young's modulus
k	material constant specifying the influence of normal stresses
k_σ	normal mean stress reduction coefficient
$k_{\tau1}$	shear mean stress reduction coefficient
$k_{\tau2}$	compound (shear and normal) mean stress reduction coefficient
m'	slope coefficient of the fatigue curve (for bending) scaled to the energy range
m_σ	slope coefficient of the fatigue curve (for bending)
m_τ	slope coefficient of the fatigue curve (for torsion)
N_f	number of cycles to failure
$W_{\sigma\varepsilon}$	stress–strain parameter
$\lvert x \rvert$	modulus of x
ε	strain
ε'_f	fatigue ductility coefficient
$\Delta\varepsilon_1$	maximum normal strain range
σ	normal stress
σ'_f	fatigue strength coefficient
σ_{af}	fatigue limit for bending
$\sigma_{H,max}$	maximum hydrostatic stress
$\sigma_{n,max}$	maximum normal stress in the critical plane
σ_{UTS}	tensile strength limit
τ	shear stress
τ_{af}	fatigue limit for torsion
τ_{max}	maximum shear stress in the critical plane

Subscripts

a	amplitude
eq	equivalent
m	mean
η	normal plane
ηa	amplitude in the normal critical plane
ηm	mean value in the normal critical plane
ηs	shear plane
ηsa	amplitude in the shear critical plane
ηsm	mean value in the shear critical plane

[16–18], Dang Van [4,16,19], McDiarmid [11,16,20], Papadopoulos [16,21–23]) for the assessment of the fatigue life of structural elements and machine components for a combination of bending and torsion, taking into account the effect of the mean value of stress and strain, because this kind of load is often encountered in practice (most often drive shafts are subjected to such loads), as well as to experimentally verify the model based on fatigue test results. The stress–strain parameter has been presented for two different methods of considering the effects of mean values. The first method is to consider the effect of the mean value in a critical plane [3], and the second one is to consider the effect of mean stress at the stage of generating of the stress and strain history.

2. Experimental data

Experimental tests were carried out on specimens of 2017A-T4 aluminum alloy (PA6 – PN) [3,15], as well as S355J0 alloy steel (18G2A – acc. to PN) [24–26] and 30NCD16 [27,28]. Strength properties of the tested materials are given in Table 1.

For 2017A-T4 aluminum alloy and S355J0 [3,15,24–26] alloy steel the results have been analyzed in conditions of plane bending, torsion and two combinations of proportional bending with torsion, for which $\tau(t) = \sigma(t)$ and $\tau(t) = 0.5\sigma(t)$ with zero and non-zero mean value (see Tables 2 and 3). For 30NCD16 [27,28] steel the stress analysis included bending, torsion, combination of proportional constant amplitude bending with torsion, for which $\tau(t) = 0.57\sigma(t)$ with a zero mean value (see Table 4). The specimens (Fig. 1 for 2017A-T4 and S355J0, Fig. 2 for 30NCD16) are cylindrical solid in the fixed zone whereas the tested zone is annular shaped.

All tests were prepared under the bending and the torque moment controlled. In all multiaxial analysis, authors used elastic model (high cycle fatigue); mean stress values and amplitudes were counted as the nominal stress.

3. Models of fatigue life

3.1. Stress models

Models based on stress used to estimate the fatigue life in the high-cycle range have been selected for comparison.

3.1.1. Goodman, Gerber, Morrow

The first group are algorithmic models that use mathematical descriptions of graphs showing dependence of change in the stress amplitude σ_a (τ_a) on mean stress values σ_m (τ_m). The following known models were used:Goodman [9]

$$\frac{\sigma_a}{\sigma_{eq}} + \frac{\sigma_m}{\sigma_{UTS}} = 1, \tag{1}$$

Gerber [8]

$$\frac{\sigma_a}{\sigma_{eq}} + \left(\frac{\sigma_m}{\sigma_{UTS}}\right)^2 = 1, \tag{2}$$

Morrow [10]

$$\frac{\sigma_a}{\sigma_{eq}} + \frac{\sigma_m}{\sigma'_f} = 1. \tag{3}$$

The extension of the foregoing fatigue models on multiaxial load cases is usually done through the adoption of a certain hypothesis that is necessary to calculate the equivalent stress and strain. The most often and widely used is the Huber–Mises hypothesis, which specifies the amplitude of the equivalent stress

Table 1
Strength properties of the tested materials.

Material (EN)	E (GPa)	σ_{UTS} (MPa)	ν	b	c	ε'_f	σ'_f (MPa)
2017(A)-T4	72	545	0.32	−0.056	−0.703	0.519	607
S355J0	213	611	0.31	−0.095	−0.448	0.126	880
30NCD16	191	1200	0.29	–	–	–	–