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Dynamic inelastic response of strain rate sensitive ductile plates due to large impact, dynamic pressure and explosive loadings

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ABSTRACT

This article studies the significant amount of literature that has been published recently on the dynamic response of plating subjected to large dynamic loadings. A theoretical method of analysis for mass impacts, dynamic pressure pulses and impulsive velocity or blast loadings on circular, square and rectangular plates is presented for the idealisation of a rigid, perfectly plastic material. This theoretical analysis caters for the influence of finite-displacements and is developed further to predict the response for plates made from a strain rate sensitive material. Relatively simple equations are presented for the maximum permanent transverse displacements which give good agreement with the corresponding experimental data, and therefore, can be used for design purposes, safety calculations, security studies, hazard assessments and forensic investigations.

1. Introduction

There has been significant recent interest in the response of ductile metal plating to both impact [1–4] and explosive pressure [5–15] loadings which produce large inelastic deformations, damage and failure. The major thrust of this work has been the generation of experimental results, development of design formulae and the validation of numerical methods of analysis. This collection of new experimental data provides an opportunity to examine the accuracy of a theoretical procedure which was published in Ref. [16] and developed for the dynamic plastic response of arbitrarily shaped plates, as discussed in Ref. [17]. In fact, most of the experimental data can be studied with this theoretical procedure. The plate materials are idealised as a rigid, perfectly plastic material and the influence of large displacements (i.e., membrane forces and geometry changes) were retained in the analysis. A theoretical solution was presented for simply supported and fully clamped rectangular plates subjected to a rectangular shaped pressure–time history, which in the limit of an infinitesimally short pressure duration, reduces to an impulsive velocity loading. Good agreement was obtained with the experimental results reported on explosively loaded fully clamped ductile metal rectangular plates. A design formula based on a simplified yield condition provided bounds on the experimental data.

Subsequently, this theoretical procedure has been used to predict the permanent transverse displacements and other features of the response for fully clamped circular plates struck by a solid mass [18]. Ref. [19] studied both circular and square plates struck by solid masses and with a boundary resisting moment mM_0, with $0 < m < 1$. The extreme cases of $m = 0$ and $m = 1$ represent simple supports and fully clamped supports, respectively. More recently, the theoretical solution was used to examine rectangular plates with a resisting moment mM_0 around the supports and struck by a large (relative to the plate mass) solid mass at the mid-span having a small footprint [4].

It is evident that theoretical solutions and design formulae are now available for a wide range of circular, square and rectangular plates subjected to impact, pressure pulses or blast loadings. The recently published experimental data, together with the previously available data, allow the predictions of the theoretical method in Ref. [16] to be assessed for accuracy and reliability for design purposes. The integration of the available information should be of value to design engineers and others who are interested in estimating the damage of structural members due to large dynamic loadings and to those seeking to validate numerical calculation methods [20].

Thus, the theoretical procedure is described briefly in Section 2 and illustrated in Section 3 for a fully clamped circular plate struck by a solid mass. The formulae for the maximum permanent transverse displacements of circular, square and rectangular plates subjected to low-velocity impacts by a solid mass,

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Notation

α_1, α_2	defined by equations (27a) and (27b)
β	aspect ratio, $\beta = B/L$
γ	mass ratio, ratio of G to the plate mass
$\varepsilon, \dot{\varepsilon}$	strain, strain rate
$\dot{\varepsilon}_e$	equivalent strain rate
η	pressure ratio, equation (A.10)
$\kappa, \dot{\kappa}$	change of curvature, rate of change of curvature
λ	dimensionless initial kinetic energy for a uniform impulsive velocity loading. Equations (A.19) and (A.22)
μ	mass per unit area of a plate
ξ_0	defined by equation (A.15c)
ρ	density of material
ρ_m	a/R, where a is radius of a cylindrical impact mass
σ_0, σ_0'	static and dynamic flow stresses
τ	duration of a rectangular pressure pulse, Fig. 2
Ω	dimensionless initial kinetic energy of an impact mass, equation (5)
m	bending moment factor at supports; $m = 0, 1$ for simply supported and fully clamped cases, respectively
p	pressure
p_c	static collapse pressure, equations (A.11), (A.13), (A.15b) and (A.17b).
p_o	magnitude of dynamic pressure
q	Cowper–Symonds exponent, equation (15).
r, θ, z	cylindrical coordinates
t	time
u	axial displacement
x, y, z	Cartesian coordinates
w	transverse displacement
$2B$	breadth of a rectangular plate
D	Cowper–Symonds coefficient, equation (15)
G	impact mass
H	plate thickness
$2L$	length of a rectangular or square plate
M	bending moment per unit length
M_o	plastic bending moment per unit length
N	membrane force per unit length
R	radius of supporting boundary of a circular plate
Q_r	transverse shear force
V_o	initial impulsive velocity
W	peak value of w.
W_f	permanent value of W.
$(\dot{X})$	$\partial(X)/\partial t$

dynamic pressure pulses and impulsive velocity loadings are presented in the Appendix A. A method for incorporating the influence of material strain rate sensitivity of the plate materials into the theoretical procedure is developed in Section 4. A comparison between the theoretical predictions and the experimental results is made in Section 5. Section 6 contains some further comments, while Sections 7 and 8 contain the discussion and conclusions.

2. Theoretical method of analysis

Now a considerable body of work has been published on the plastic collapse behaviour of ductile structures when subjected to a wide range of static and dynamic loadings which produce an inelastic material response. However, the theoretical formulations, in most cases, are based on governing equations using the first order or classical theory, which was developed for structures undergoing infinitesimal displacements. In the case of beams and plates, for example, the response can change significantly under transverse loadings due to geometry changes brought about by sufficiently large loadings causing finite-displacements. A smaller literature has examined the large transverse deflection response of structures which caters for this phenomenon, but this paper presents some results for the dynamic behaviour of plates obtained using the method reported in Ref. [16].

The analysis developed in Refs. [16,17] uses the general equation of motion for an arbitrarily shaped plate (includes a beam as a special case) which is integrated with the aid of Green's theorem and leads to a set of equations expressing energy conservation. In other words, the external work rate (impact, dynamic pressure and inertia forces) is equated to the internal energy dissipated at plastic hinges and within plastic zones. The plates have a uniform thickness, H, and are made from a rigid, perfectly plastic material and transverse, in-plane and rotational inertia, as well as transverse shear effects, are retained in the basic equations. The equations which emerge from this general method can be simplified for particular problems, such as axisymmetry studied in Ref. [18], or by neglecting transverse shear effects (e.g. Ref. [4]). In other cases, the response of a particular problem might involve plastic dissipation at only straight line kinematically admissible hinges with the remainder of a plate remaining rigid, as in Ref. [4] for rectangular and square plates.

The theoretical analysis in Refs. [16,17] can be written in the following form when neglecting in-plane and rotational inertia

$$-G\ddot{W}\dot{W} - \int_A \mu\ddot{w}\dot{w}\,\mathrm{d}A + \int_A p\dot{w}\,\mathrm{d}A = \int_A \{(M_r + wN_r)\dot{\kappa}_r + (M_\theta + wN_\theta)\dot{\kappa}_\theta\}\mathrm{d}A + \sum_{m=1}^{n} \times \int_{C_m} (M_r + wN_r)(\partial\dot{w}/\partial r)_m\,\mathrm{d}C_m + \sum_{u=1}^{\nu} \int_{C_u} Q_r(\dot{w})_u\,\mathrm{d}C_u \tag{1}$$

where G is an impact mass, and μ is the mass per unit surface area of a plate. The transverse displacement of a plate is w, while $\dot{w}$ and $\ddot{w}$ are the associated velocity and acceleration. W is the transverse displacement of the plate which is immediately underneath a striking mass.

The terms on the left hand side of equation (1) are the work rate due to the pressure pulse, p, and the inertia forces of the mass G and plate mass, where A is the surface area of a plate. The first term on the right hand side of equation (1) is the energy dissipated in any continuous deformation fields. The second term gives the energy dissipated in n plastic hinges, each having an angular velocity $(\partial\dot{w}/\partial r)_m$ across a hinge of length C_m. The final term is the plastic energy absorption in ν transverse shear hinges, each having a velocity discontinuity $(\dot{w})_u$ and a length C_u. Equation (1) ensures that the external work rate equals the internal energy dissipation.